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An Application of MILP-based Block Planning in the Chemical Industry

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Abstract Production processes in the chemical industry often consist of an initial stage in which a basic chemical product is produced and a subsequent stage which produces the final product variants according to demand forecasts or customer orders. Typically, a filling and packaging process is integrated into this final production stage. For scheduling the production and packaging activities a mixed-integer linear programming (MILP) model is proposed which is based on the definition of setup families and the production of product variants in a pre-defined sequence. The model determines the size and the time phasing of the individual production lots. In contrast to conventional lot-sizing and scheduling models, the proposed optimization model is based on a continuous representation of time.

Keywords Production planning and scheduling – Lot sizing – Mixed-integer linear programming model – chemical industry

1 Introduction

Considering the production of pharmaceuticals as an example, two major production stages can be distinguished. In the initial stage a basic chemical product is produced through a number of time-consuming chemical processing steps while in the subsequent stage the basic chemical product is processed further and the final product variants are produced according to demand forecasts or customer orders. In the chemical and pharmaceutical industry, different variants of a product are achieved by adjusting the process parameters and the mix of input materials as well as using a variety of packaging formats according to the end users' requirements and country-specific regulations.

In this paper we focus on the final production and packaging stage of the entire chemical production process which often consists of a number of filling and packaging lines. This combined production and packaging process typically constitutes the bottleneck in the final stage of the production system. Such production systems can be found in many branches of the process industries, e.g. in the chemical or consumer goods industries.

Specifically in the chemical and pharmaceutical industry, parallel lines are established for each package type, e.g. liquid and solid drugs. A line usually produces a number of product types and packages them into individual units for shipment to customer warehouses. Each product type corresponds to a specific recipe which determines the ingredients and the processing conditions while product variants are defined by the specific packaging form. Typically, production lines are set up for a specific product type requiring a major setup such that the changeover between the individual product variants can be accomplished with only a minor setup or cleaning operation.

The intention of this paper is to develop an optimization model for short-term scheduling and lot sizing in single-stage chemical production. The model formulation is based on the block planning concept proposed by Günther et al. (2006). The remainder of this article is organized as follows. In the next section the major characteristics of the block planning concept are explained. In the subsequent Section 3, a block planning model based on mixed-integer linear programming is developed. Finally, conclusions are drawn and the practical applicability of the proposed modelling approach is discussed.

2 Block planning

In short-term production planning, decisions have to be made on lot sizes and their timing and sequencing on the various manufacturing and packaging lines. In particular, in process-related industries there is often a natural sequence in which the various products are to be produced in order to minimize total changeover time and to maintain product quality standards. For example, setups are sequenced from products with high to low purity requirements or from the brighter color of a product to the darker. Hence, families of products can be identified which are produced in a given sequence under the same basic equipment setup. In this case, major setups are incurred for changing over between product families while only minor setups are needed for switching to another product within the same family.

Since conventional lot-sizing and scheduling models do not sufficiently reflect the conditions given in process industries we propose an alternate approach, called block planning, for scheduling production orders on a continuous time scale with demand elements being assigned to distinct delivery dates. Moreover, issues like definition of setup families with consideration of major and minor setup times and multiple non-identical production lines with dedicated product-line assignments are addressed in a realistic way. The model formulation aims at supporting operative decisions in a single-stage production system.

So far, in the scientific literature the application of block planning concepts and the development of corresponding optimization models are widely neglected though block planning concepts are easy to implement and reflect managerial practice prevalent in make-and-pack production. Block planning approaches have been developed for specific applications, e.g. hair dye production (cf. Günther et al. 2006) or in fresh food industries (cf. Lütke Entrup et al. 2005). Recently, an integrated model for production and distribution planning in the fast moving consumer goods industry has been presented by Bilgen and Günther (2010).

In the chemical industry application investigated here, specific combined filling and packaging lines are used for the various packaging types, e.g. liquid and solid forms of drugs. This makes it possible to subdivide the entire planning problem into line-specific sub-problems. Hence, in the following we consider only one single line. For the filling and packaging line at hand, the block pattern given in Fig. 1 illustrates the sequence of product types which are produced each after a major setup.

Once the line is set up this way, a family of different product variants is processed in the pre-defined sequence each with a production sub-lot of variable size. When changing over from one product variant to another, a minor setup time is incurred. Further blocks are appended in a cyclical fashion until the end of the planning horizon is reached.

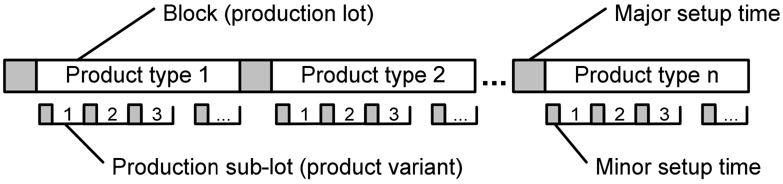

Figure 1: Block pattern for a production line.

The major characteristics of the basic block planning concept upon which the development of the MILP model in the next section is based can be summarized as follows.

- Given the assignment of products to setup families, fixed setup sequences of products are defined based on human expertise and technological requirements. Each block corresponds to a setup family and is scheduled in a cyclical fashion.
- The composition of the individual blocks is not necessarily the same. Binary decision variables indicate whether a product is set up or not and continuous decision variables reflect the lot size of each product in the block. Depending on the development of demand over time, the lot sizes of an individual product may vary from block to block. As a result, also the time needed to complete a block is variable.
- The start-off and completion dates of a block are not directly linked to the period boundaries. Hence, a block is allowed to start in an earlier period as soon as the predecessor block has been completed. However, the execution of a block must be finished before the end of the assigned period.
- Typically, a major setup operation is performed before starting or after completing a block (e.g. for cleaning the manufacturing equipment), while only a minor setup operation is required when changing between products within the same block (e.g. for provision of material or for adjusting the processing conditions).
- Macro-periods, e.g. weeks, are used for the assignment of blocks while the assignment of external demand elements is based on micro-periods, e.g. days.
- For each product inventory balances are updated on a periodic basis according to the production output and the given external demand.
- The usual objective function is to minimize total inventory holding and setup costs. Major constraints arise from the available production capacities and the satisfaction of external demand.

3 Model formulation

In this paper, a flexible block planning approach based on a mixed-integer linear programming (MILP) model is proposed which introduces a considerable degree of flexibility for determining the length of an entire block, varying the production quantities for individual products variants within a block, and for scheduling the start-off and completion times of all blocks and production lots. The MILP model formulation uses a continuous time representation to model the production runs for all products within a setup family while a discrete time representation is used for the assignment of blocks to macro-periods and for the daily assignment of demand elements. Binary decision variables refer to the setup of blocks, production lots for packaging forms, and sub-lots for product types. Continuous variables are used to model the lot sizes and sub-lot sizes, inventory levels and the timing of the various production activities. The objective function minimizes major setup costs for production lots, minor set up costs for sub-lots, and inventory holding costs. The notation used in the model formulation is given as follows.

Indices and sets

$i \in I$ blocks $i = 1,...,last(I)$

$i \in I^{fix}$ fixed blocks ($i = 1,...,last(I^{fix})$)

$i \in I^{opt}$ optional blocks ($i = first(I^{opt}),...,last(I^{opt})$)

$p \in P$ products

$p \in P(i)$ products which are produced in block i

$p \in P(j)$ products which belong to product family j

$j \in J$ product families

$t \in [\underline{\alpha}_i, \overline{\alpha}_i]$ periods with $\underline{\alpha}_i$ and $\overline{\alpha}_i$ indicating the earliest start and latest feasible completion time, respectively, of block i

Parameters

$\underline{ls}_p$ minimum sub-lot size of product p in the fixed block

M_p maximum sub-lot size of product p

a_p unit production time for product p

s_p minor setup time per sub-lot of product p

S_i major setup time for block i

d_{pt} external demand of product p assigned to the end of period t

c_p^{min} minor setup cost per sub-lot of product p

c_i^{maj} major setup cost for block i

c_p^{inv} inventory holding cost per unit of product p per period

Decision variables and domains

$x_{ip} \geq 0$ sub-lot size of product p in block i

$y_{ij} \in \{0,1\}$ =1, if product family j is assigned to optional block i

$\rho_{ip} \in \{0,1\}$ =1, if product p is set up in block i (0, otherwise)

$\sigma_i \in \{0,1\}$ = 1, if optional block i is active, i.e. a product family is assigned to it (0, otherwise)

$z_{it} \in \{0,1\}$ =1, if block *i* has been finished up to the end of period *t* (0, otherwise)

$q_{pt} \geq 0$ output of product *p* available at the end of period *t*

$F_{pt} \geq 0$ inventory level of product *p* at the end of period *t*

$\alpha_i \geq 0$ start time of block *i*

$\Omega_i \geq 0$ end time of block *i*

$\delta_i \geq 0$ duration of block *i*

The following MILP model formulation is based on the distinction of *fixed* and *optional* blocks and does not rely on a strict block to period assignment. Since each product family must be scheduled at least once in the course of the planning horizon – given that a positive net-requirement for at least one item within the product family exists – we consider the setup of one block of each product family as being *fixed*, but still leave the start-off time of the block open. Further setups of a product family are considered *optional* because the number and the size of the respective blocks as well as their timing have to be determined by the optimization model. In order to obtain the sequence of the fixed blocks, the corresponding product families are sorted in ascending order of their run-out-times. The second part of the production schedule contains the optional blocks. As a flexible and practice-oriented approach we propose that a "menu" of optional blocks is defined by the human planner leaving open the assignment of product families to blocks and the sub-lot sizes of the individual products. Since not all of the allowed optional blocks must be utilized, *active* and *non-active* blocks can be distinguished. The constraints of the block planning model are the following.

Setup constraints for fixed blocks: Product families whose stocks deplete during the planning horizon require at least one setup, i.e. the value of the corresponding binary setup variable is set to one in constraint (1) and, according to constraint (2), the sub-lot sizes of all products belonging to the product family must meet the minimum lot size necessary to cover demand over the pre-determined run-out time of the initial stock.

$$\rho_{ip} = 1 \qquad \forall i \in I^{fix}, p \in P(i) \,\big|\, \underline{ls}_p > 0 \tag{1}$$

$$x_{ip} \geq \underline{ls}_p \qquad \forall i \in I^{fix}, p \in P(i) \,\big|\, \underline{ls}_p > 0 \tag{2}$$

Setup constraints for optional blocks: According to (3) block *i* can only be active, if its predecessor block *i-1* is active. In other words, non-active blocks are assigned to the end of the schedule. Constraint (4) ensures that exactly one product family is assigned to an optional block, if the block is active, and no product family is assigned, if the block is not active. According to (5) binary setup variables for the production sub-lots are allowed to take values of one only if the respective product family *j* is assigned to the block. Constraint (6) models the relationship between the size of the sub-lot and the binary setup variable. The size of the sub-lot is enforced to zero if no corresponding setup operation is performed.

$$\sigma_i \leq \sigma_{i-1} \qquad \forall i \in I^{opt} \text{ with } \sigma_{last(I^{fix})} = 1 \tag{3}$$

$$\sum_{j \in J} y_{ij} = \sigma_i \qquad \forall i \in I^{opt} \qquad (4)$$

$$\sum_{p \in P(j)} \rho_{ip} \le y_{ij} \cdot |P(j)| \quad \forall i \in I^{opt},\ j \in J \qquad (5)$$

$$x_{ip} \le M_p \cdot \rho_{ip} \qquad \forall i \in I^{opt} \qquad (6)$$

Block schedule: Constraint (7) models the duration of a block which results from the major setup time for the block, the minor setup times for all sub-lots and the time required for producing the sub-lot sizes. According to (8) a block is allowed to start as soon as the predecessor block has been completed. Constraints (9) and (10) impose time windows for the earliest start-off and for the latest completion time of a block. Constraint (11) defines the end time of block *i*.

$$\delta_i = \sum_{p \in P} \left(s_p \cdot \rho_{ip} + a_p \cdot x_{ip} \right) + S_i \cdot \sigma_i \quad \forall i \in I \qquad (7)$$

$$\alpha_i \ge \alpha_{i-1} + \delta_{i-1} \qquad \forall i = 2,...,last(I) \text{ with } \alpha_1 = 0 \qquad (8)$$

$$\alpha_i \ge \underline{\alpha}_i \qquad \forall i \in I \qquad (9)$$

$$\alpha_i + \delta_i \le \overline{\alpha}_i \qquad \forall i \in I \qquad (10)$$

$$\Omega_i = \alpha_i + \delta_i \qquad \forall i \in I \qquad (11)$$

Matching production output and demand: Since the assignment of product families to blocks is not given in advance, additional decision variables and constraints are introduced in order to trace the completion of production activities over time and to match the resulting production output against external demand. While a continuous time representation is used to schedule the production runs for all products within a product family, demand elements are assigned to the end of a period. So-called heavy-side variables (cf. Grunow et al., 2003) are introduced which indicate if block *i* has been finished up to a particular period *t*. The following logical constraints (12) and (13) enforce the heaviside variables to zero for all periods prior to the completion period of the block and to one for the completion period and all succeeding periods. For all periods *t* prior to the completion period of block *i*, the right-hand side in (12) is greater than zero and less than one and in (13) less than zero. Hence the binary heaviside variables are enforced to zero. For the remaining periods the right-hand side in (12) is greater than one and in (13) between zero and one. As a result, the heaviside variables are enforced to one. This way the completion of a production sub-lot is indicated by a switch from zero to one in the periodic development of the heaviside variables.

$$z_{it} \le 1 + \frac{t - \Omega_i}{\overline{\alpha}_i} \qquad \forall i \in I, t = \underline{\alpha}_i,...,\overline{\alpha}_i \qquad (12)$$

$$z_{it} \ge \frac{t - \Omega_i}{\overline{\alpha}_i} \qquad \forall i \in I, t = \underline{\alpha}_i,...,\overline{\alpha}_i \qquad (13)$$

The following constraints are needed to derive the quantities of final products from the various optional blocks depending on the assigned product family. According to (14) and (15) only at the period, in which the values of the heaviside variables switch from zero to one, output from the production process is available. For all other periods the respective variables takes a value of zero. Constraint (16) makes sure that the daily output quantities match the production sub-lot size. Constraint (17) contains the inventory balances.

$$q_{pt} \leq M_p \cdot z_{it} \qquad \forall i \in I, p \in P, t = \underline{\alpha}_i \tag{14}$$

$$q_{pt} \leq M_p \cdot \left(z_{it} - z_{i,t-1} \right) \quad \forall i \in I, p \in P, t = \underline{\alpha}_i + 1, \ldots, \bar{\alpha}_i \tag{15}$$

$$\sum_{t=\underline{\alpha}_i}^{\bar{\alpha}_i} q_{pt} = x_{ip} \qquad \forall i \in I, p \in P, t = \underline{\alpha}_i, \ldots, \bar{\alpha}_i \tag{16}$$

$$F_{pt} = F_{p,t-1} + q_{pt} - d_{pt} \quad \forall p \in P, t \in T \quad \text{with} \quad F_{p0} = given \tag{17}$$

Objective function: The entire optimization model consists of constraints (1) to (17) and the objective function (18) stated below. The objective function aims to minimize total costs comprised of major and minor setup costs and inventory holding costs.

$$\min \sum_{i \in I^{opt}} c_i^{maj} \cdot \sigma_i + \sum_{i \in I} \sum_{p \in P} c_p^{min} \cdot \rho_{ip} + \sum_{p \in P} \sum_{t \in T} c_p^{inv} \cdot F_{pt} \tag{18}$$

4 Conclusions

In this paper an MILP-based block planning approach for production scheduling in single-stage production systems has been presented. The size of the MILP model depends on the number of product families and specific product variants, the density of the time grid which is imposed for the assignment of demand elements, and on the number of blocks in the schedule. Despite its general applicability, a key factor which has a considerable impact on the computational effort is the introduction of daily time periods and the use of heaviside variables which are needed to trace the completion date of the production lots. Especially, when a large number of small-sized demand elements have to be considered, the computational complexity of the model solution is accordingly increased and CPU times rise significantly. Nevertheless, initial numerical tests showed that optimal solutions can be obtained within a few minutes of CPU time.

In any case, the framework of mixed-integer linear programming provides considerable flexibility for the integration of application-specific features which is not given for conventional lot sizing and scheduling models known from the academic literature. Combined with the human planner's expertise on the definition of setup families and the natural sequence in which the various products are to be produced in order to minimize total changeover time and to prevent quality losses, block planning represents a practical and efficient way to support decision makers in practice.

References

[1] Bilgen B, Günther H-O (2010) Integrated production and distribution planning in the fast moving consumer goods industry: A block planning application. OR Spectrum (to appear)

[2] Grunow M, Günther H-O, Yang G (2003) Plant coordination in pharmaceutics supply

networks. OR Spectrum, 25: 109–141

[3] Günther H-O, Grunow M, Neuhaus U (2006) Realizing block planning concepts in make-and-pack production using MILP modelling and SAP APO©. International Journal of Production Research, 44: 3711-3726

[4] Lütke Entrup M, Günther H-O, van Beek P, Grunow M, Seiler T. (2005) Mixed integer linear programming approaches to shelf-life-integrated planning and scheduling in yoghurt production. International Journal of Production Research, 43: 5071-5100