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Generalized Weber Model for Hub Location of Air Cargo

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Abstract Many of air cargo companies adapt the hub-and-spokes system where economies of scale exist in the transportation cost. In this paper, we formulate economies of scale using nonlinear cost function of distances and demands, and analyze how the change of the transportation cost affects single hub location of air cargo using the generalized Weber problem on the regular demand points of one and two-dimensional region and the actual location of airport in East Asia. The optimal location will move to the largest demand points as the economies of scale in distances become larger or the economies of scale in demands become smaller. We confirm economies of scale in both distances and demands from the parameters estimated by the actual transportation cost in East Asia and find the optimal hub location of air cargo.

Keywords Hub location; Weber problem; Air cargo; Transportation cost; Economies of scale

1 Introduction

Many of airlines adapt the hub-and-spokes system where economies of scale exist in the transportation cost. Most of the air cargo airlines have a single hub airport in United States while most of passenger airlines have a few hub airports. O'Kelly[7] formulated the hub location problem of airline using the Weber model and showed that the optimal point locates in mid-west area of United States that many hub of air cargo initially located. Watanabe et al.[9] analyzed how the change of the transportation cost affects single hub location of air cargo in the United States using the generalized Weber problem. Recent literatures mainly formulate as the discrete facility location problem using the mathematical programming model, but the Weber model can be locatable in continuous plane.

The Weber problem is famous for Weberian location triangles in the Industrial location theory as shown in Fig.1, and is to find a location of a facility which minimizes the weighted sum of Euclidean distances to a set of demand points[6]. This mini-sum point is usually called as the Weber point and there is a minimum demand condition where Weber point is the absorbed solution[8] which locate on the exist demand point. Recently, effective algorithm for the generalized Weber problem has developed[3]. There are many researches whose transportation cost is nonlinear functions with respect to the Euclidean distances, but demands are also need to be nonlinear functions.

In addition, analyzing the change of the transportation cost is very important because the transportation cost has been greatly influenced by the sudden rise in the fuel price in this decade. There are volume-related rates and distance-related rates in tranportation[2], so it's important to formulate transportation cost of air cargo using nonlinear function of demand and distance[4]. We treat the air cargo rates as the transportation cost and will estimate the transportation cost function from actual rates.

In this research, we are to find a location of a hub airport which minimizes the weighted sum of transportation cost which is power function of the Euclidean distances and demands to a set of demand points. We analyze how the change of the transportation cost affects a single hub location on regular demands in one and two-dimensional region. As case study, we apply this model to find the optimal location of hub using the actual handling data of air cargo in East Asia.

2 Model Description

We formulate as continuous single-facility location problem using nonlinear cost function which the marginal cost decreases as distance or demands increases.

Let $P_1,...,P_n$ be distinct demand points when n is the total number of airports. Let q_i be the demand of i-th demand point. The coordinates of the demand point is (x_i, y_i) and that of the optimal point is (x, y). K is the coefficient of the fixed cost and α is the elasticity of demand and β is the elasticity of distance. Objective function is the total transportation cost C and may be formulated as follows:

$$\min_{x,y} C = K \sum_{i=1}^{n} {q_i}^{\alpha} l_i(x,y)^{\beta} \quad (1)$$

where $l_i(x,y) = \sqrt{(x-x_i)^2 + (y-y_i)^2}$ is the Euclidean distance between (x,y) and (x_i, y_i).

We will explain basic feature of Eq.(1) using Fig.2. When α is equal to one, we get linear function of demand. If α is less than one, we get concave function, otherwise convex function. This is the same for β. We can explain original Weber problem when bpth α and β is equal to one.

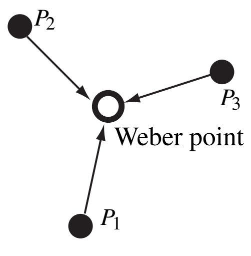


Figure 1: Weber problem

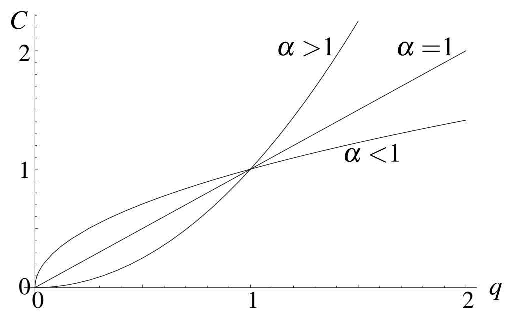


Figure 2: Nonlinear function

3 Analysis on Regular point

3.1 One-dimensional linear region with two demand points

At first, we consider the linear region whose length is L with two demand points A and B on both ends. Let the demand on the point A and B is q_1 and q_2 respectively, and q_2 is larger than q_1. From Eq.(1), we can formulate as follows:

$$\begin{aligned} \min_x C &= K\sum_{i=1}^{2} {q_i}^{\alpha} l_i(x)^{\beta} \\ &= K\{{q_1}^{\alpha}x^{\beta} + {q_2}^{\alpha}(L-x)^{\beta}\}. \end{aligned} \tag{2}$$

After solving the first derivative of this expression with respect to x, we can get the optimal location as follows:

$$x^* = \frac{L}{1+(q_1/q_2)^{\frac{\alpha}{\beta-1}}}. \tag{3}$$

We substitute $L = 10000$ for lentgh and $q_1 = q$, $q_2 = 2q(q = 10000)$ for the parameters of demands. We can plot the ordinate of optimal point in Fig.3. The optimal point moves to the center of line as α become smaller, if β is greater than one. On the other hand, the optimal point is on the largest demand point B, if β is less than one.

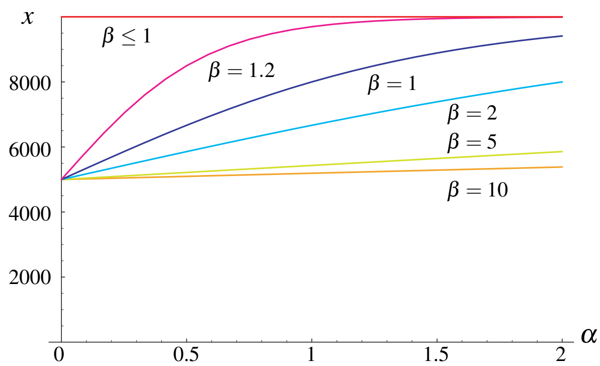


Figure 3: Optimal point in one-dimensional linear region

3.2 Two-dimensional triangle region with three demand points

Next, we consider the triangle region whose length of each sides is L with three demand points on each vertices A, B and C. The coordinates of point A, B and C become (0,0), (0,L) and (L/2,$\sqrt{3}L/2$) respectively. Let the demand on point A, B and C is q_1, q_2 and q_3 respectively, and q_3 is larger than q_1 and q_2.

Generally speaking, we can't solve Eq.(1) in this condition analytically except the case of $\beta = 2$. In this case, we can formulate as follows:

$$\begin{aligned}\min_{x,y} C &= K\sum_{i=1}^{3} q_i{}^{\alpha} l_i(x,y)^2 \\ &= K[q_1{}^{\alpha}(x^2+y^2)+q_2{}^{\alpha}\{(x-L)^2+y^2\} \\ &\quad +q_3{}^{\alpha}(x-L/2)^2+(y-\sqrt{3}L/2)^2]. \end{aligned} \tag{4}$$

After solving the first derivative of this expression with respect to x and y, we can get the optimal location as follows:

$$x^* = \frac{2q_2{}^{\alpha}+q_3{}^{\alpha}}{q_1{}^{\alpha}+q_2{}^{\alpha}+q_3{}^{\alpha}}\frac{L}{2}, \tag{5}$$

$$y^* = \frac{q_3{}^{\alpha}}{q_1{}^{\alpha}+q_2{}^{\alpha}+q_3{}^{\alpha}}\frac{\sqrt{3}L}{2}. \tag{6}$$

When q_1 is equal to q_2, the x-coordinate of optimal point become $x^* = L/2$ and optimal point locate on the bisector of base.

We substitute $L = 10000$ for lentgh and $q_1 = q$, $q_2 = q$, $q_3 = 2q(q = 10000)$ for the parameters of demands. We can plot the ordinate of optimal point on the bisector of base in Fig.5. The optimal point gradually moves to the interior of the triangle as α become smaller, if β is greater than one. This result is same as the case of one-dimensional region. On the other hand, the optimal point suddenly moves to the interior of the triangle, if both of α and β is less than one.

We calculate the contour lines of the objective function of Eq.(1) and find the optimal point. From the comparison between (i) and (ii) in Fig.4, the optimal point moves to the largest demant point C as *beta* become smaller. From the comparison between (i), (iii) and (iv) in Fig.4, the optimal point moves to the center of gravity as α become smaller.

4 Case study of air cargo in East Asia

4.1 Demands and Parameters

We treat ten major airports in East Asia including Hong Kong, Incheon and Narita as shown in Fig.6. We use the annual handling data of international air cargo[1] as demand data q_i. We define the Euclidean distance on the map of Mercator projection as distance $l_i(x,y)$.

We confirm the economies of scale in both distances and demands from the parameters estimated by the actual rate of air cargo. JFFI[5] reported the matrix of rate which consists of the weights and the distances in 2007. We also include data of United States because many integrators of United States also operate in East Asia. We estimate parameters of α, β and K of Eq.(1) by the nonlinear regression using least squares method, and the results are table1. As R-square coefficient of determination is very high, the regression almost perfectly fits the data.

Both of the elasticity of demands α and the elasticity of distances β is less than one in almost all countries, so we can conclude that there are economies of scale in demands and distances. The results of α and β in China are very close to those in United States.

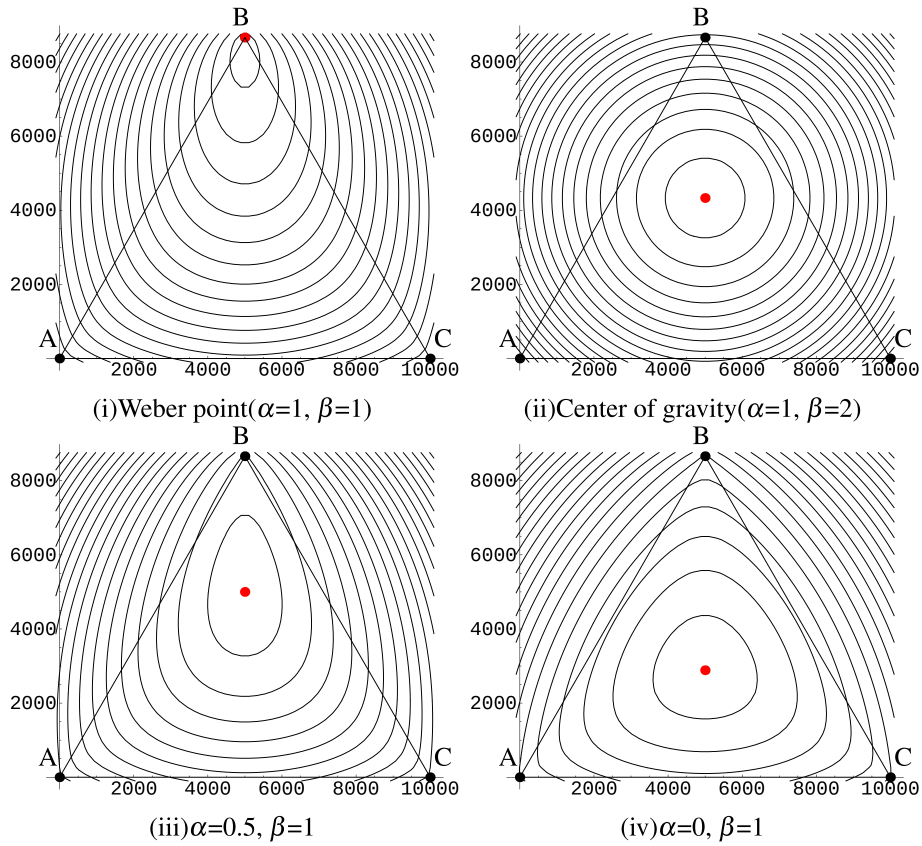


Figure 4: Iso-cost curves in two-dimensional triangle region

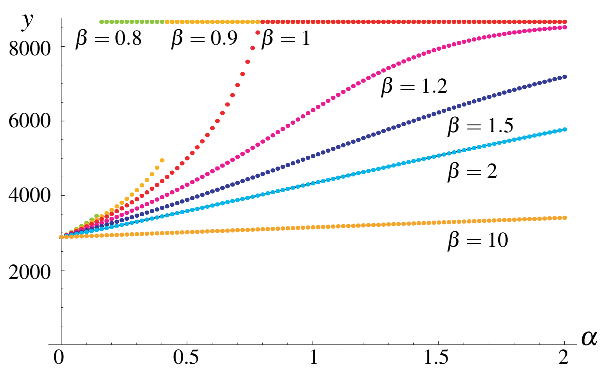


Figure 5: Optimal point in two-dimensional triangle region

Table 1: Result of estimated parameters

Country	K	α	β	R^2
Japan	1.726	0.894	0.750	0.997
United States	147.704	0.957	0.195	0.999
China	48.102	0.903	0.221	0.987
Korea	1.266	0.377	1.001	0.815

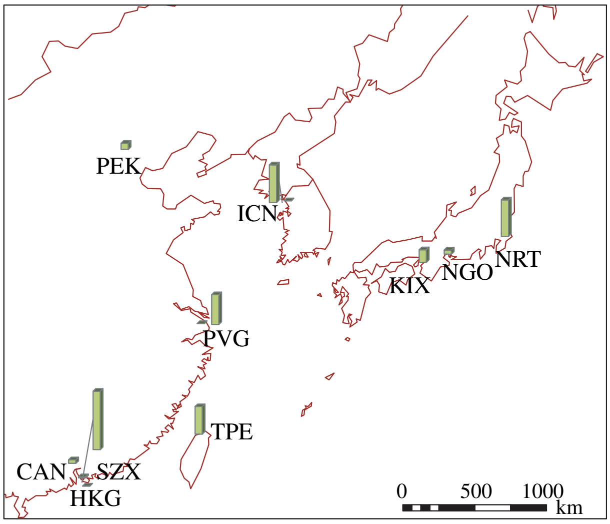


Figure 6: Distribution of demand

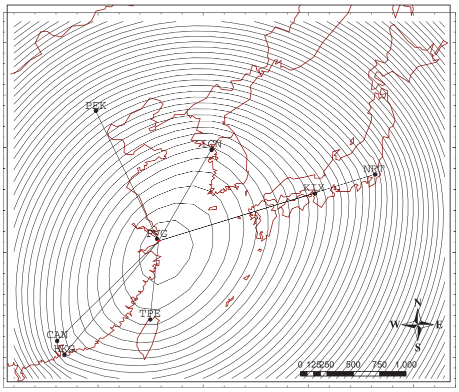
4.2 Computational results

We calculate the contour lines of the objective function of Eq.(1) in Fig. 7 and find the optimal point.

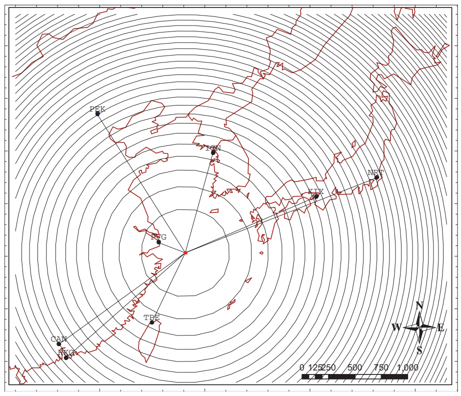
Weber point is located on the East China Sea and is very close to Shanghai in China. Center of gravity is also located on the East China Sea, but is located more east than Weber point.

Optimal point of Japan is absorbed solution on Shanghai airport. Optimal point of Korea is the same as that of Japan. Optimal point of United States is absorbed solution on Hong Kong airport which have the largest demand. Optimal point of China is same as that of United States.

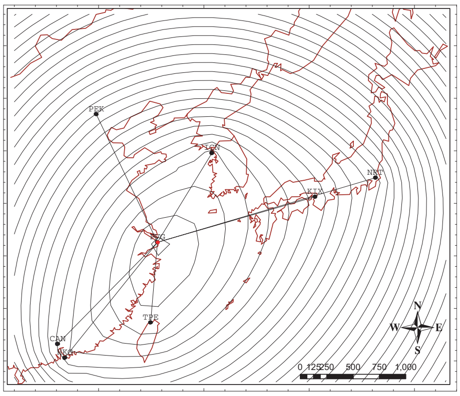
The iso-cost curves of Weber point and center of gravity are convex functions and have only one global optimal solution because β is not less than one. On the contrary, the iso-cost curves of every counties except Korea are not convex fuctions and have many local optimal solutions because β is less than one. The iso-cost curves of United States and China are very complex compared with those of Japan and Korea because β is close to zero.



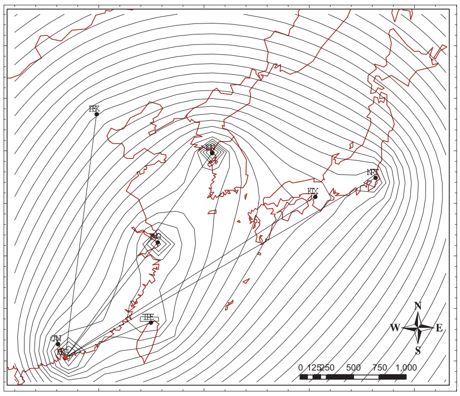
(i)Weber point(α=1, β=1)



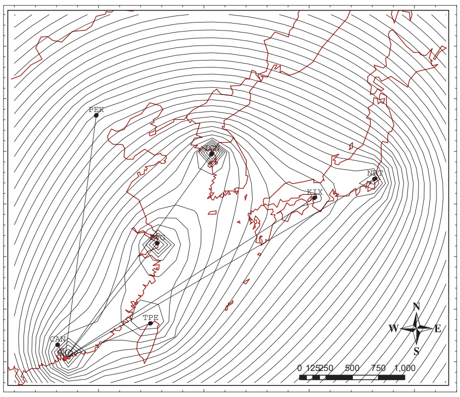
(ii)Center of gravity(α=1, β=2)



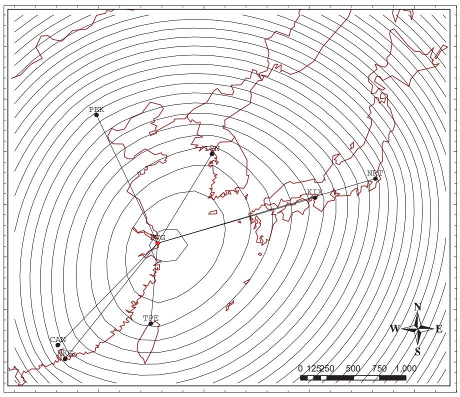
(iii)Japan



(iv)United States



(v)China



(vi)Korea

Figure 7: Iso-cost curves in East Asia

5 Conclusion

In this paper, we formulate the economies of scale using nonlinear cost function of distances and demands, and analyze how the change of the transportation cost affects single hub location of air cargo using the generalized Weber problem on the regular demand points of one and two-dimensional region and the actual location of airport in East Asia. The optimal location moves to the largest demand points as the economies of scale in distances become larger or the economies of scale in demands become smaller. We confirm economies of scale in both distances and demands from the parameters estimated by the actual transportation cost in East Asia and find the optimal hub location of air cargo.

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