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A Public Patentee's Technology Transfer under a Leadership Structure*

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Abstract In this paper, we consider the issue of optimal licensing from the viewpoint of a public patent holder having no production facilities when the product market has a Stackelberg leadership structure. We show that fee licensing is always at least as good as royalty licensing for the public patentee maximizing the social welfare. It is also shown that for small innovations, there exists an equilibrium in which the public patentee licenses his patented technology to only the efficient, profit-maximizing private firm but he never licenses his technology to only the inefficient private firm.

Keywords Game theory; Subgame perfect equilibrium; Public patentee; Leadership structure; Licensing

1 Introduction

Since the seminal work of Arrow [1], a rich and diverse literature on licensing has been developed. Kamien and Tauman [3] is one of the most important studies in the field of technology transfer through licensing. They set up a model where an independent private patentee having no production facilities decides which licensing policy (fee or royalty) to choose and to which firm(s) to transfer his patented technology to gain the most profit in a Cournot market. They show that an independent private patentee always prefers fee licensing to royalty licensing. Whether a patentee prefers fee or royalty licensing in this situation has been a major topic of interest since. Kabiraj [2] considers a situation where a patent holder is a non-producer and the product market has a leadership structure with one leader and one follower competing in quantities. He points out that no other literature has discussed this situation so far, though this problem is of practical importance. He shows that, for small innovations, royalty licensing is optimal for a private patentee, while for relatively large innovations, fee licensing dominates royalty licensing.

Based on Kabiraj [2], this paper constructs a basic model in which an independent public patent holder with a cost-reducing technology licenses his patented technology to Stackelberg oligopolistic private firms. One difference from the model of Kabiraj [2]

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in our model is the existence of a public patentee who maximizes the social welfare. Although many examples exist in Japan, such as the National Institute of Advanced Industrial Science and Technology (AIST) and RIKEN, little attention has been paid to the case in which an independent patent holder is public. The purpose of this paper is to investigate what licensing policy a public patent holder would decide and to whom the patent would be given. We show that fee licensing is always at least as good as royalty licensing for the public patentee. We also show that, for small innovations, there exists an equilibrium in which the public patentee licenses his patented technology to only the efficient firm, but he never licenses to the inefficient firm.

2 Basic Model

We consider a duopoly producing a homogeneous product. Let $N \equiv \{1,2\}$ be the set of private firms. The product market is characterized by a Stackelberg leadership structure with firm 1 as the leader and firm 2 as the follower. The market price p is determined by $p(q_1,q_2) = \max(0, a - q_1 - q_2)$, where a represents the maximum price of the product and q_i is a firm i's production level. Firm 1 produces at the constant unit cost c_L and firm 2 produces at the constant unit cost c_H. Throughout this paper, we assume that $0 < c_L < c_H < a$ and $a + 2c_L - 3c_H > 0$, i.e., firm 1 is the more efficient firm and, with their old technology, both firms produce positive output levels in the Stackelberg-Nash equilibrium.

A patentee, who has no production facilities, is a state-owned public firm; thus, his object is to maximize the social welfare. The public patentee has a patented technology that can reduce any licensee's unit cost of production by the amount ε, where $0 < \varepsilon < c_L$.[1] Consider the situation in which the public patentee can license his patented technology to firms by a policy of either a (lump-sum) fee or a (per-unit) royalty. Under the fee policy, the licensee pays a lump-sum fee f (≥ 0) for licensing. On the other hand, if the royalty policy is adopted, the licensee pays at a uniform royalty rate r, $0 \leq r \leq \varepsilon$, per unit of production.

The game is organized as follows.

The first stage: The public patentee determines which licensing policy (fee or royalty) he should choose to maximize the social welfare.
The second stage: The public patentee announces the amount of a fee (or a royalty rate) to all firms in the market to license his patented technology.
The third stage: Each firm simultaneously and independently decides whether to buy the license or not.
The fourth stage: After the set of licensees becomes commonly known, firm 1 chooses its quantity to maximize its profit.
The fifth stage: After observing firm 1's quantity, firm 2 chooses its quantity to maximize its profit.

The equilibrium concept is the subgame perfect Nash equilibrium. The game is solved by backward induction in the following section.

[1] The magnitude of ε and the difference between the two costs are the key parameters for the public patentee to choose his policy on technology transfer. This is because these parameters change the behavior of each private firm in the market.

3 Stackelberg-Nash Equilibrium

We begin by considering fourth and fifth stages in which firms 1 and 2 have unspecified constant unit production costs of c_1 and c_2, respectively. Results of this model will serve as a reference for deriving equilibria under each licensing policy studied later.

At the fifth stage, given firm 1's quantity q_1, firm 2's profit maximization problem is as follows:

$$\max_{q_2 \geq 0} \pi_2(q_1, q_2) = (a - q_1 - q_2 - c_2)q_2.$$

Denote the solution of this problem by $R_2(q_1)$, called the reaction function of firm 2. At the fourth stage, we have the following firm 1 profit maximization problem:

$$\max_{q_1 \geq 0} \pi_1(q_1, q_2) = (a - q_1 - q_2 - c_1)q_1,$$
$$\text{subject to } q_2 = R_2(q_1).$$

Let $A = a - 2c_1 + c_2$, $B = a + 2c_1 - 3c_2$ and $C = a + c_1 - 2c_2$. Solving the above maximization problems, the Stackelberg-Nash equilibrium quantities are

$$(q_1, q_2) = (A/2, B/4) \quad \text{if } A > 0 \text{ and } B > 0 \tag{1a}$$
$$(q_1, q_2) = (a - c_2, 0) \quad \text{if } C > 0 \text{ and } B \leq 0 \tag{1b}$$
$$(q_1, q_2) = ((a - c_1)/2, 0) \quad \text{if } C \leq 0 \tag{1c}$$
$$(q_1, q_2) = (0, (a - c_2)/2) \quad \text{if } A \leq 0, \tag{1d}$$

and their profits are

$$(\pi_1, \pi_2) = \left(A^2/8, B^2/16\right) \quad \text{if } A > 0 \text{ and } B > 0 \tag{2a}$$
$$(\pi_1, \pi_2) = ((c_2 - c_1)(a - c_2), 0) \quad \text{if } C > 0 \text{ and } B \leq 0 \tag{2b}$$
$$(\pi_1, \pi_2) = \left((a - c_1)^2/4, 0\right) \quad \text{if } C \leq 0 \tag{2c}$$
$$(\pi_1, \pi_2) = \left(0, (a - c_2)^2/4\right) \quad \text{if } A \leq 0. \tag{2d}$$

The boundary condition on the market price yields the above four patterns of the equilibrium quantities.

Depending on whether each firm is a licensee or not, there are four cases: (i) no firm is a licensee (superscript *NL*), (ii) both firms are licensees (*AL*), (iii) only the more efficient firm is a licensee (*LL*), and (iv) only the less efficient firm is a licensee (*LH*).

4 Fee Licensing

We first consider fee licensing. In fee licensing, the social welfare under the fee policy is given by

$$SW = \pi_1 + \pi_2 + \frac{(q_1 + q_2)^2}{2}.$$

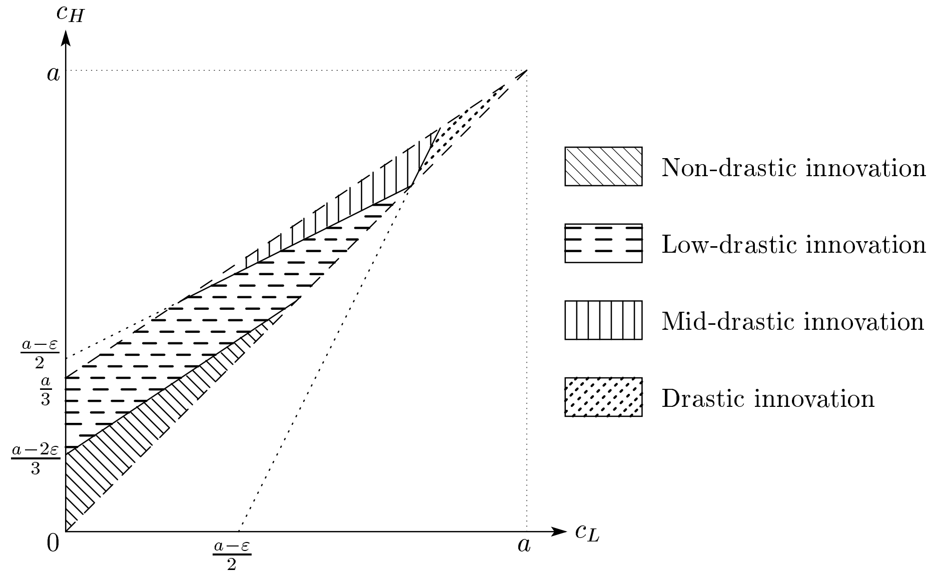

Figure 1: The region for each innovation type

If the public patentee licenses to both firms, the two firms' quantities and profits, and the social welfare are

$$(q_1^{AL}, q_2^{AL}) = \left(\frac{a-2c_L+c_H+\varepsilon}{2}, \frac{a+2c_L-3c_H+\varepsilon}{4}\right), \tag{3}$$

$$(\pi_1^{AL}, \pi_2^{AL}) = \left(\frac{(a-2c_L+c_H+\varepsilon)^2}{8}, \frac{(a+2c_L-3c_H+\varepsilon)^2}{16}\right), \text{ and} \tag{4}$$

$$SW^{AL} = \frac{1}{32}\{15a^2+28(c_L-\varepsilon)^2+23(c_H-\varepsilon)^2- 20a(c_L-\varepsilon)-36(c_L-\varepsilon)(c_H-\varepsilon)-10a(c_H-\varepsilon)\}, \tag{5}$$

respectively. Similarly, if he licenses to neither firm, both firms' quantities, profits and the social welfare are

$$(q_1^{NL}, q_2^{NL}) = \left(\frac{a-2c_L+c_H}{2}, \frac{a+2c_L-3c_H}{4}\right),$$

$$(\pi_1^{NL}, \pi_2^{NL}) = \left(\frac{(a-2c_L+c_H)^2}{8}, \frac{(a+2c_L-3c_H)^2}{16}\right), \text{ and}$$

$$SW^{NL} = \frac{1}{32}\left\{15a^2+28c_L^2+23c_H^2-20ac_L-36c_Lc_H-10ac_H\right\}, \tag{6}$$

respectively. In these cases, both firms always supply the product, because of the assumptions in Section 2. Comparing (5) and (6), we have $SW^{AL} > SW^{NL}$. If the public patentee sets the fee $f \le \min(\pi_1^{AL}-\pi_1^{LH}, \pi_2^{AL}-\pi_2^{LL})$, both firms buy the patent at the third stage. Hence, the public patentee always licenses his patented technology regardless of the magnitude of the innovation.

Next, we consider the situation in which only one firm is licensed. We need to consider four separate cases: non-drastic ($\varepsilon < (a+2c_L-3c_H)/2$), low-drastic ($(a+2c_L-3c_H)/2 \leq \varepsilon < a+c_L-2c_H$), mid-drastic ($a+c_L-2c_H \leq \varepsilon < a-2c_L+c_H$) and drastic innovations ($a-2c_L+c_H \leq \varepsilon$). Figure 1 depicts the c_L, c_H regions for each innovation case.

4.1 Non-drastic innovation ($\varepsilon < (a+2c_L-3c_H)/2$)

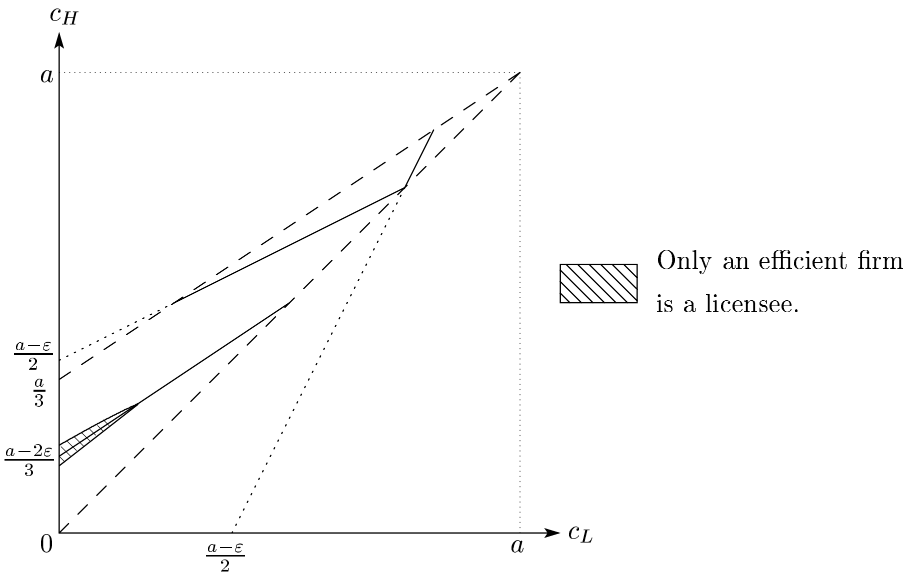

Figure 2: The equilibrium region in which the public patentee gives his technology to only the efficient firm ($\varepsilon \leq 16a/53$)

In this case, even when the public patentee licenses to only firm 1 (2), firm 2 (1) is not driven out of the market. So, when only firm 1 is licensed, substituting $c_1 = c_L - \varepsilon$ and $c_2 = c_H$ into (1a) and (2a) gives the following firms' equilibrium quantities and profits:

$$(q_1^{LL}, q_2^{LL}) = \left(\frac{a-2c_L+c_H+2\varepsilon}{2}, \frac{a+2c_L-3c_H-2\varepsilon}{4} \right) \text{ and}$$
$$(\pi_1^{LL}, \pi_2^{LL}) = \left(\frac{(a-2c_L+c_H+2\varepsilon)^2}{8}, \frac{(a+2c_L-3c_H-2\varepsilon)^2}{16} \right).$$

Similarly, substituting $c_1 = c_L$ and $c_2 = c_H - \varepsilon$ into (1a) and (2a), we have

$$(q_1^{LH}, q_2^{LH}) = \left(\frac{a - 2c_L + c_H - \varepsilon}{2}, \frac{a + 2c_L - 3c_H + 3\varepsilon}{4} \right) \text{ and} \tag{7}$$

$$(\pi_1^{LH}, \pi_2^{LH}) = \left(\frac{(a - 2c_L + c_H - \varepsilon)^2}{8}, \frac{(a + 2c_L - 3c_H + 3\varepsilon)^2}{16} \right) \tag{8}$$

in the case in which only firm 2 is a licensee. Therefore, the social welfare in each case is

$$SW^{LL} = \frac{1}{32} \left\{ 15a^2 + 28(c_L - \varepsilon)^2 + 23c_H^2 - 20a(c_L - \varepsilon) - 36(c_L - \varepsilon)c_H - 10ac_H \right\} \text{ and}$$

$$SW^{LH} = \frac{1}{32} \left\{ 15a^2 + 28c_L^2 + 23(c_H - \varepsilon)^2 - 20ac_L - 36c_L(c_H - \varepsilon) - 10a(c_H - \varepsilon) \right\}. \tag{9}$$

We next consider which firm should be licensed from the viewpoint of the public patentee. Comparing SW^{AL} with SW^{LH} and SW^{LL}, we have

$$SW^{LH} - SW^{AL} = \frac{\varepsilon}{8}(-5a + 14c_L - 9c_H + 2\varepsilon) \tag{10}$$

$$SW^{LL} - SW^{AL} = \frac{\varepsilon}{32}(-10a - 36c_L + 46c_H + 13\varepsilon). \tag{11}$$

With a non-drastic innovation, (10) is negative. So, the public patentee tries not to license to only firm 2. On the other hand, (11) can be positive. If the public patentee sets the fee $\pi_2^{AL} - \pi_2^{LL} \leq f \leq \pi_1^{LL} - \pi_1^{NL}$, only firm 1 buys the patent at the third stage. Therefore, we have the following lemma, which can be confirmed in Figure 2.

Lemma 1.
For non-drastic innovations, the public patentee licenses his patented technology to only firm 1 if $(10a + 36c_L - 46c_H)/13 < \varepsilon < (a + 2c_L - 3c_H)/2$*. Otherwise, the patented technology diffuses to both firms. If* $16a < 53\varepsilon$*, there does not exist a case in which the public patentee licenses his patented technology to only firm 1.*

4.2 Low-drastic innovation ($(a + 2c_L - 3c_H)/2 \leq \varepsilon < a + c_L - 2c_H$)

With a low-drastic innovation, when the public patentee licenses to only firm 2, firm 1 can supply the product. So, the firms' equilibrium quantities and profits are (7) and (8), and the social welfare is (9). Firm 2, on the other hand, is driven out of the market when the public patentee licenses to only firm 1, but firm 1 cannot gain the monopoly profit. Substituting $c_1 = c_L - \varepsilon$ and $c_2 = c_H$ into (1b) and (2b), we have

$$(q_1^{LL}, q_2^{LL}) = (a - c_H, 0) \text{ and}$$
$$(\pi_1^{LL}, \pi_2^{LL}) = ((c_H - c_L + \varepsilon)(a - c_H), 0).$$

Therefore, the social welfare is

$$SW^{LL} = \frac{1}{2} \left\{ a^2 - 2a(c_L - \varepsilon) + 2(c_L - \varepsilon)c_H - c_H^2 \right\}.$$

Comparing each social welfare, we have $SW^{AL} > SW^{LH}$, as in the case of non-drastic innovation, and

$$SW^{LL} - SW^{AL} = \frac{1}{32}\big(a^2 - 12ac_L - 28c_L^2 + 10ac_H + 68c_Lc_H - 39c_H^2 + 2a\varepsilon + 20c_L\varepsilon - 22c_H\varepsilon - 15\varepsilon^2\big). \quad (12)$$

With a low-drastic innovation, there exists a case in which (12) can be positive, namely, the case $16a > 53\varepsilon$. If the public patentee sets the fee $\pi_2^{AL} - \pi_2^{LL} \leq f \leq \pi_1^{LL} - \pi_1^{NL}$, only firm 1 buys the patent at the third stage. Therefore, we have the following lemma.

Lemma 2.
For low-drastic innovations, there exists a case in which the public patentee licenses his patented technology to only firm 1, namely, the case $16a \geq 53\varepsilon$. *Otherwise, he always licenses to both firms.*

4.3 Mid-drastic innovation ($a + c_L - 2c_H \leq \varepsilon < a - 2c_L + c_H$)

As in the case of non-drastic innovations, firms' quantities and profits are (7) and (8), and the social welfare is (9) when the public patentee licenses to only firm 2. Because, in this case, firm 1 can gain the monopoly profit when only firm 1 is licensed, substituting $c_1 = c_L - \varepsilon$ and $c_2 = c_H$ into (1c) and (2c) gives the following firms' quantities and profits:

$$(q_1^{LL}, q_2^{LL}) = \left(\frac{a - c_L + \varepsilon}{2}, 0\right) \text{ and} \quad (13)$$

$$(\pi_1^{LL}, \pi_2^{LL}) = \left(\frac{(a - c_L + \varepsilon)^2}{4}, 0\right). \quad (14)$$

Therefore, the social welfare is

$$SW^{LL} = \frac{3(a - c_L + \varepsilon)^2}{8}. \quad (15)$$

By (3), (4), (13) and (14), $\pi_1^{AL} > \pi_1^{LL}$, $\pi_2^{AL} > \pi_2^{LL}$ and $q_1^{AL} + q_2^{AL} > q_1^{LL} + q_2^{LL}$. Therefore, $SW^{AL} > SW^{LL}$. Furthermore, we have $SW^{AL} > SW^{LH}$, as in the case of non-drastic innovations. Hence, we have the next lemma.

Lemma 3.
For mid-drastic innovations, the public patentee always licenses his patented technology to both firms.

4.4 Drastic innovation ($a - 2c_L + c_H \leq \varepsilon$)

In this case, if the public patentee licenses to only firm 1, firm 2 is driven out of the market. Therefore, firms' quantities and profits are (13) and (14), and the social welfare is (15). On the other hand, when the public patentee licenses to only firm 2, firm 1 is driven out of the market. So, substituting $c_1 = c_L$ and $c_2 = c_H - \varepsilon$ into (1d) and (2d), we have

$$(q_1^{LH}, q_2^{LH}) = \left(0, \frac{a - c_H + \varepsilon}{2}\right) \text{ and}$$

$$(\pi_1^{LH}, \pi_2^{LH}) = \left(0, \frac{(a - c_H + \varepsilon)^2}{4}\right),$$

and the social welfare is

$$SW^{LH} = \frac{3(a - c_H + \varepsilon)^2}{8}.$$

Obviously, $SW^{LL} > SW^{LH}$ by (15), and $SW^{AL} > SW^{LL}$ for the same reason as in the case of mid-drastic innovation. Therefore, we have the next lemma.

Lemma 4.
For drastic innovations, the public patentee always licenses his patented technology to both firms.

Lemmas 1, 2, 3 and 4 directly give the following proposition, which can be confirmed in Figure 2.

Proposition 1.
Under the fee policy, if $16a \geq 53\varepsilon$*, there exists a case in which the public patentee licenses his patented technology to only firm 1. Otherwise, the public patentee always licenses his patented technology to both firms.*

5 Royalty Licensing

Consider royalty licensing. Obviously, if $r \leq \varepsilon$, both firms accept the offer, because $c_L - \varepsilon + r \leq c_L$ and $c_H - \varepsilon + r \leq c_H$. Therefore, substituting $c_1 = c_L - \varepsilon + r$ and $c_2 = c_H - \varepsilon + r$ into (1a) and (2a), we have

$$(q_1^{AL}, q_2^{AL}) = \left(\frac{a - 2c_L + c_H + \varepsilon - r}{2}, \frac{a + 2c_L - 3c_H + \varepsilon - r}{4} \right) \text{ and}$$
$$(\pi_1^{AL}, \pi_2^{AL}) = \left(\frac{(a - 2c_L + c_H + \varepsilon - r)^2}{8}, \frac{(a + 2c_L - 3c_H + \varepsilon - r)^2}{16} \right).$$

Under the royalty policy, the social welfare is given by $SW = \pi_1 + \pi_2 + (q_1 + q_2)^2/2 + r(q_1 + q_2)$ when both firms are licensed. Therefore, the social welfare is

$$SW^{AL} = \frac{1}{32}\{15a^2 - 20ac_L + 28c_L^2 - 10ac_H - 36c_Lc_H + 23c_H^2 + (30a - 20c_L - 10c_H + 15\varepsilon)\varepsilon + 2r(-3a + 2c_L + c_H - 3\varepsilon) - 9r^2\}. \quad (16)$$

Maximising (16) with respect to r subject to the constraint that $0 \leq r \leq \varepsilon$, we have the solution $r^* = 0$, because $a > c_H > c_L > \varepsilon > 0$. Therefore, we have the following proposition.

Proposition 2.
Under the royalty policy, the public patentee always licenses his patented technology to both firms at the royalty rate $r = 0$*.*

Finally, we compare the fee policy and the royalty policy. By (5) and (16) with $r = 0$, the equilibrium social welfare in royalty licensing is equal to that in fee licensing to both firms. There, however, exists a case in which licensing to only firm 1 is socially preferred under the fee policy. Therefore, we have the following proposition.

Proposition 3.
Fee licensing is at least as good as royalty licensing for the public patentee.

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