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Mathematical Model and Solving Method Based on Software for the Shortest Time Limited Transportation Problem

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Abstract The shortest time limited transportation problem and its extension case are studied in this paper. By introducing the 0-1 variables, the shortest time limited transportation problem can be formulated into a linear programming model while its extension case can be formulated into a nonlinear programming model. The programs for solving both models based on Lingo software are compiled. The models and algorithms are tested by two examples. The results show that the solving method base on software is both effect and exact, so it is an efficient method for solving large size of real problems.

Keywords model and algorithm, linear programming; the shortest time limited; transportation problem; Lingo software

1 Introduction

There is a special transportation problem in reality. This problem is often related to the urgent materials transportation, such as rescue equipment, equipment used for dealing with emergency, people or medical treatment things, and the fresh food with short storage period. In the process of finding the transport scheme, the most important thing to be considered is the shortest time other than the minimum total cost. For example, when the nature disaster happens, in order to rescue the people besieged in the floodwater as soon as possible, we need to transport some doctors and nurses from several hospitals to different disaster sites. In this process, the benefit resulting from saving time is much more important than the benefit resulting from saving part of transportation cost. There are several earthquakes in the world in these two or three years. The earthquake damages our life seriously. When earthquake happens, the time is all, so saving time is the most important issue when we transport people or equipment to the earthquake sites.

The shortest time limited transportation problem is proposed in such background [1]. In the past few years, several scientists have studied this problem. On the one hand, various types of algorithms for solving this problem were proposed, such as the solving

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method based on the graph [1], the solving method based on the table [2][3], the network solving method base on Ford-Fulkerson max-flow algorithm [4][5], the dynamic programming algorithm [6] and so on. On the other hand, various extension of the shortest time limited transportation problem and their algorithms are investigated, such as the shortest time limited minimum cost transportation problem [7],Multi-objective shortest time limited transportation problem based on both the security and the time factors [8], the shortest time limited transportation problem with the transportation capacity constraint [9], the general shortest time transportation problem with the transportation time being a function of the transportation quantity in the corresponding road [10] [11], and so on.

Although the mathematical models of the shortest time limited transportation problem and its extension are proposed in literatures [1], [10], [11], the objective functions of these models are not obvious functions of the decision variables, hence the objective functions are not simple maximum or minimum functions, they are minmax functions. These models are not the canonical linear or nonlinear programming models. On the other hand, all the solving algorithms for the shortest time limited transportation problems or its extension problems have not been executed by compiling softwares. With the size of the problems becoming larger and larger, it will be very difficult to finish the compute process by hand. So it is the best way to find the optimal solution of the large size problem in the aid of software.

In this paper, after investigating the characters of the shortest time limited transportation problems, the mathematical programming models are constructed by introducing a set of 0-1 variables. Furthermore, the software for solving this models are compiled. Simulations have been done on the examples from literatures.

2 The Linear Programming Model of the Shortest Time Limited Transportation Problem

The shortest time limited transportation problem can be described as: Given m provide areas $A_1, A_2, \cdots, A_m$ of some material, the output of A_i is a_i $(i = 1,2,\cdots,m)$; n demand areas $B_1, B_2, \cdots, B_n$, the demand of B_j is b_j $(j = 1,2,\cdots,n)$. The transportation time from A_i to B_j is t_{ij}. The problem is: How to organize the transport scheme, such that the longest time used for transporting materials from A_i $(i = 1,2,\cdots,m)$ to B_j $(j = 1,2,\cdots,n)$ is minimum.

This problem was described by Li [1] as: Finding a set of x_{ij} $i = 1,2,\cdots,m; j = 1,2,\cdots,n$, such that the $\max(t_{ij}|x_{ij} > 0)$ is minimum in the following constraint conditions

$$s.t. \begin{cases} \sum_{j=1}^{n} x_{ij} = a_i & i = 1,2,\cdots,m \\ \sum_{i=1}^{m} x_{ij} = b_j & j = 1,2,\cdots,n \\ x_{ij} \geq 0, & i = 1,2,\cdots,m, j = 1,2,\cdots,n \end{cases}$$

Where x_{ij} is the amount of materials transported from A_i to B_j.

Since the objective function is not a linear function of the decision variables, so the model is not a linear programming model . Noticing that the objective function is a minmax function, by introducing a set of 0-1 variables, the model can be reformulated into a linear programming model.

Define the 0-1 variables as follows

$$z_{ij} = \begin{cases} 1 & x_{ij} > 0 \\ 0 & x_{ij} = 0 \end{cases}$$

Then the shortest time limited transportation problem can be formulated into a linear programming model.

$$\begin{array}{ll} \min z = y & (1.1) \\ s.t. \begin{cases} \sum_{j=1}^{n} x_{ij} = a_i & i = 1,2,\cdots,m \quad (1.2) \\ \sum_{i=1}^{m} x_{ij} = b_j & j = 1,2,\cdots,n \quad (1.3) \\ y \geq t_{ij} z_{ij} & i = 1,2,\cdots,m; j = 1,2,\cdots,n \quad (1.4) \\ x_{ij} \leq M z_{ij} & i = 1,2,\cdots,m; j = 1,2,\cdots,n \quad (1.5) \\ x_{ij} \geq 0, z_{ij} = 0,1 & i = 1,2,\cdots,m; j = 1,2,\cdots,n \quad (1.6) \\ y \geq 0 & (1.7) \end{cases} \end{array} \qquad (1)$$

Where the objective function (1.1) minimizes the upper bound of time used in all transportation road from $A_i, (i = 1,2,\cdots,m)$ to $B_j, (j = 1,2,\cdots,n)$, which means that the time for finishing the transportation task is minimum. Constraints (1.2) means that the amount of supply must be transported to the demand sites. Constraints (1.3) means that the amount of demand of an area equals to the sum of its accepted materials. Constraint (1.4) express that y is the upper bound of every road's transportation time. Constraint (1.5) indicates that when $x_{ij} > 0$, z_{ij} must be equal to 1. Constraints (1.6) and (1.7) are the descriptions of the variables's value.

Obviously, model (1) is a linear programming model, which can be solved by software.

3 The Mathematical Programming Model of the Extension Shortest Time Limited Transportation Problem

In the basic model of the shortest time limited transportation problem, the transportation time t_{ij} from A_i to B_j is a constant, which does not change with x_{ij} changes. In reality, x_{ij} often affects the transportation time. For example, the more amount of material a truck load, the longer time it needs to load and unload the material, the more slower the truck's speed is, and the longer time it needs to finish the transportation task. So t_{ij} is usually the function of x_{ij}. The authors of reference [10] have studied the relationship between the amount of load and the real transportation time. They let the real transportation time be the sum of two parts. One part is t_{ij} which is a constant related to the distance between A_i and B_j. The other part is the added time which includes the load and unload time, and can be expressed by a function of x_{ij}. The authors of paper [10] discussed the case when the function is a linear function $\alpha_{ij} x_{ij}$ (where α_{ij} is determined by the type of truck, the type of road between A_i and B_j). In this case, the real total transportation time between A_i

and B_j is $t_{ij}+\alpha_{ij}x_{ij}$. This is the extension of the basic model. When $\alpha_{ij}=0$, this model turn to be the basic model. Based on the same idea, the authors of paper [10] investigated the case when the added time is a nonlinear function of x_{ij}. Suppose that the added time is a quadratic function of x_{ij}, for example, the added time equals to $\alpha_{ij}x_{ij}^2$, then the real total transportation time is $t_{ij}+\alpha_{ij}x_{ij}^2$. For both extension cases of the shortest time limited transportation problem, an iterative solving method based on max-flow algorithm is proposed respectively in [10] and [11]. Both methods are complex in computing and can not guarantee to find the global optimal solution in short time. Furthermore, the authors did not introduce the execute process by compile these algorithms' software.

Noticing the characters of the extension shortest time limited transportation, by introducing a set of 0-1 variables, the extension problem can be formulated into a nonlinear programming model. When the reality transportation time is a quadratic function of x_{ij} [11], the corresponding extension problem can be formulated into the following nonlinear programming model (2). The other extension case in paper [10] can be formulated into a linear programming model similar to model (2).

$$\begin{aligned}
&\min z = y && && (2.1)\\
&s.t.\begin{cases}
\sum_{j=1}^{n} x_{ij} = a_i & i=1,2,\cdots,m \quad (2.2)\\
\sum_{i=1}^{m} x_{ij} = b_j & j=1,2,\cdots,n \quad (2.3)\\
y \geq t_{ij}z_{ij}+\alpha_{ij}x_{ij}^2 & i=1,2,\cdots,m; j=1,2,\cdots,n \quad (2.4)\\
x_{ij} \leq Mz_{ij} & i=1,2,\cdots,m; j=1,2,\cdots,n \quad (2.5)\\
x_{ij} \geq 0, z_{ij}=0,1 & i=1,2,\cdots,m; j=1,2,\cdots,n \quad (2.6)\\
y \geq 0 & (2.7)
\end{cases}
\end{aligned} \qquad (2)$$

The constraints and variables in model (2) have the similar means as those in model (1).

The results of paper [10] and [11] give us a new idea for investigating the extension shortest time limited transportation problem. In reality, the relationship between the added transportation time and the transportation amount may be more complicated. The function relationship may not be the simple linear or quadratic function, furthermore it may not have an obvious expression formulate. So it is necessary to investigate the more general extension shortest time limited transportation problem.

Suppose that the added transportation time from A_i to B_j is $\alpha_{ij}f(x_{ij})$, the total transportation time is $t_{ij}+\alpha_{ij}f(x_{ij})$. Then the general extension shortest time limited transportation problem can be formulated into the following mathematical model.

$$\begin{aligned}
&\min z = y && && (3.1)\\
&s.t.\begin{cases}
\sum_{j=1}^{n} x_{ij} = a_i & i=1,2,\cdots,m \quad (3.2)\\
\sum_{i=1}^{m} x_{ij} = b_j & j=1,2,\cdots,n \quad (3.3)\\
y \geq t_{ij}z_{ij}+\alpha_{ij}f(x_{ij}) & i=1,2,\cdots,m; j=1,2,\cdots,n \quad (3.4)\\
x_{ij} \leq Mz_{ij} & i=1,2,\cdots,m; j=1,2,\cdots,n \quad (3.5)\\
x_{ij} \geq 0, z_{ij}=0,1 & i=1,2,\cdots,m; j=1,2,\cdots,n \quad (3.6)\\
y \geq 0 & (3.7)
\end{cases}
\end{aligned} \qquad (3)$$

The constraints and variables in model (3) have the similar means as those in model (1) and (2).

Model (3) is the most general model, model (1) and (2) are respectively the special cases of model (3).

Since the models (1)(2)(3) are all canonical mathematical programming models, so they can be solved by softwares. It is easy to compile an easily operational visual program software. When using it, one only needs to input the data and variables information, click the button, then he can obtain the optimal solution. In the following section, we will use Lingo software to compile the algorithms' programs for solving the mathematical models, and do simulations on two examples from references [1] and [11] respectively.

4 Computational Results

We use two examples from papers [1] and [11]. Using Lingo software to solve the mathematical models, we obtain the optimal solutions of these two examples and compare the results with that in papers [1] and [11].

Example 1 [1] After a flood disaster happened in a large scope of some areas. A type of epidemic appeared in five denizen sites, the doctors are needed to be transported from three hospitals A_1,A_2,A_3 to these five sites $B_1,B_2,\cdots,B_5$ for curing the sicks. The number of doctors every site demand, the number of doctors every hospital can supply, and the transfer time from each hospital to every site are listed in table 1. How to arrange the transportation scheme such that all the doctors can get to the sites in the shortest time?

Table 1 Number of doctors supply and demand, and the transfer time from each hospital to every site.

	B_1	B_2	B_3	B_4	B_5	supply
A_1	3	7	5	4	5	11
A_2	9	5	6	6	2	13
A_3	5	10	8	7	5	8
demand	3	8	5	10	6	

This is the basic shortest time limited transportation problem, which can be formulated into model (1). Using Lingo software to solving model (1) with the corresponding data, we can find the optimal solution listed in table 2.

Table 2 The optimal solution of example 1, the number in each pane corresponds to the number of doctors transported from A_i $(i=1,2,3)$ to B_j $(j=1,2,\cdots,5)$.

	B_1	B_2	B_3	B_4	B_5	supply
A_1	0	0	5	6	0	11
A_2	0	8	0	4	1	13
A_3	3	0	0	0	5	8
demand	3	8	5	10	6	

The shortest time to finish the transfer task is 6. The optimal transportation scheme is sending 5 doctors from A_1 to B_3, 6 doctors from A_1 to B_4, 8 doctors from A_2 to B_2, 4 doctors from A_2 to B_4, 1 doctors from A_2 to B_5, 3 doctors from A_3 to B_1, 5 doctors from A_3 to B_5. This result is the same as that in paper [1].

Example 2 After the earthquake happened in some area. Two villages B_1,B_2 were damaged seriously. Some succor materials need to be transported from 3 cities A_1,A_2,A_3

to two villages B_1, B_2. The amount of supply , demand and the basic time t_{ij} from city A_i to village B_j are listed in table 3. Suppose that the added transportation time from A_i to B_j is a quadratic function of transportation amount x_{ij}, the coefficient α_{ij} is listed in table 4.

Table 3 Supply , demand, and the basic transfer time t_{ij}.

	B_1	B_2	supply
A_1	23	30	4
A_2	26	29	3
A_3	25	22	3
demand	4	6	

Table 4 Coefficient α_{ij}.

	B_1	B_2
A_1	2	3
A_2	2	2
A_3	4	2

In paper [11], the authors found the amount of transport materials and the reality transportation time in every road from A_1, A_2, A_3 to B_1, B_2 using the dichotomy search method based on network flow algorithm. The results are listed in table 5 and 6.

Table 5 The amount of transport materials obtained by the authors of paper [11].

	B_1	B_2
A_1	2.25	1.45
A_2	1.1	1.9
A_3	0.35	2.65

Table 6 The reality transport time in every road obtained by authors of paper [11]

	B_1	B_2
A_1	36.005	36.3075
A_2	28.42	36.22
A_3	25.49	36.045

The results in table 6 show that the shortest time to finish the transfer task according to the scheme in table 5 is 36.3075, which is the time from A_1 to B_2. Since all the reality time in the other road is less than 36.3075, we can easily adjust the transfer scheme by closed road method to reduce the reality transfer time from A_1 to B_2, so the objective function of this problem can be decreased. This means the solution in paper [11] is not the optimal solution of example 2.

By using model (2) and Lingo software, we can easily find the optimal solution of example 2. The optimal solution is listed in table 7 and table 8.

Table 7 The optimal transportation solution obtained by model (2) and Lingo software

	B_1	B_2
A_1	2.562253	1.437747
A_2	1.102456	1.897544
A_3	0.335291	2.664709

Table 8 The reality transport time in every road obtained by by model (2) and Lingo software.

	B_1	B_2
A_1	36.13028	36.20135
A_2	24.30818	36.20135
A_3	25.44968	36.20135

The results in table 8 show that the shortest time for finishing the transportation scheme is 36.20135. Since the reality time in three roads to B_2 are the same, so the result can not be improved by adjusting the transport amount in the roads to B_2, which means that the results in table 7 and table 8 are the optimal solutions of example 2.

Compare the results in table 6 and table 8, we find that the result in table 6 is not the optimal solution while the result in table 8 is the global optimal solution. This means that it is a very good strategy to solve the shortest time limited transportation problem and its extension cases by software.

5 Conclusion

The characters of the shortest time limited transportation problem and its extension are investigated in this paper. By skillfully introducing a set of 0-1 variables, the shortest time limited transportation problem and its extension are formulated into mathematical programming models. The relationship among three models are analyzed, and the software for solving these programming model based on Lingo are complied. The simulations are done on examples of paper [1] and [11], the results show that the solving method based on software is both time saving and easily to find the optimal solution.

With the development of computer technology, it is becoming an absolutely necessarily part of management to solve the reality problem based on computer software. Especially for the large size of complicated problem. The idea of this paper can be used to solve the similar problems.

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