The Ninth International Symposium on Operations Research and Its Applications (ISORA'10)
Chengdu-Jiuzhaigou, China, August 19–23, 2010

Optimal Production-Inventory Policy under Energy Buy-Back Program

Xiao-Song Ding Ji-Hong Zhang* Xi Chen Wei Shi

International Business School, Beijing Foreign Studies University, Beijing, 100089

Abstract This paper proposes a production-inventory model with setup cost for production and financial compensation for stopping production when the buy-back program is activated, which is a modified model to that of [Chen et al 2007] by including the setup cost. Under an energy buy-back program, we consider $M+1$ types of market scenarios and the corresponding buy-back levels with different financial compensations determined by the specific supply-demand condition. We show that the optimal production-inventory policy is of an (s,S) type for all market scenarios. The inclusion of setup cost in the proposed model may better depict the real-world scenario and help the manufacturers make more reasonable decisions.

Keywords dynamic programming; production-inventory model; (s,S) policy; energy buy-back program; setup cost; financial compensation

1 Introduction

Soaring power transactions between utilities caused by regulatory and operational changes in major developed countries led to a huge popularity of energy buy-back programs in the last decade; see [Coy 1999, Wald 2000], etc. In [Chen et al 2007], the authors studied the production-inventory problem in which the manufacturer participates in the aforementioned energy buy-back program, which gives participating manufacturers financial compensations for reducing their energy use when it is activated. They have shown that a base-stock policy is optimal for normal (non-peak) market condition whereas the (s,S) policy is optimal for peak market conditions. However, one of the simplification of their model is the exclusion of setup cost that is fairly common in the real-world practice in production. Other relevant work can be found in [Beyer et al 2006, Chao and Chen 2005, Sethi and Cheng 1997, Song and Zipkin 1993], and so on.

In this paper, a modified model is proposed by including the setup cost as well as financial compensations for participating the buy-back program. Taking into the consideration of setup cost for production better depicts the real-world scenario since certain amount of setup costs incur for almost all the manufacturers whenever production happens. Under the buy-back program, we consider $M+1$ types of market scenarios and the corresponding buy-back levels with different financial compensations determined by the specific supply-demand condition.

*Email: zhangjihong@bfsu.edu.cn

With the modified model proposed in this paper, we show that the optimal production-inventory policy is of an (s,S) type for all market scenarios (both non-peak and peak states). For any period k with $M+1$ different states, if the inventory level is at or above s, the manufacturer should participate the buy-back program and stop production; if the inventory level is below s, the manufacturer should reject the offer and produce up to S. Within certain period k, those s for different states i, denoted by $s_k^{(i)}$, are supposed to be with different values. We will show that under some assumptions, the relationship among the reproduction levels for different states satisfies

$$s_k^{(0)} \geq s_k^{(1)} \geq s_k^{(2)} \geq \ldots \geq s_k^{(M)}$$

whereas the order-up-to level, denoted by S_k, remains the same for all market scenarios.

The following section formulates the general model. The third section characterizes the optimal production-inventory policy through induction, and the final section concludes this paper.

2 Model

We consider $M+1$ market scenarios including one non-peak state and M types of peak states. We define $L_0 = 0$ for the non-peak state $i = 0$. The meaning seems obvious. Within a non-peak state, the manufacture will not receive any reward even if he decides to stop production. We also define a financial compensation, denoted by L_i, with $L_i > 0$ for $i = 1,\ldots,M$, corresponding to the buy-back level for each peak state i. Apart from the financial compensation, we introduce a constant setup cost $K > 0$ for production, i.e., the cost will be increased by K whenever the manufacturer decides to begin production at the beginning of each period. Then we consider a multi-period production-inventory model in which ξ_k, $k = 1,\ldots,N$, are independent and identically distributed with mean value μ, the cumulative distribution function $\Phi(\cdot)$ and density function $\phi(\cdot)$ for the single period demand. A linear production cost with unit production cost c and a convex and coercive holding/shortage cost function $G(y)$, that is, as $|y| \to +\infty$, $G(y) \to +\infty$, are also assumed. Moreover, there is no production-capacity constraint. Let p_i denote the corresponding discrete probability distribution regarding L_i with $\sum_{i=0}^{M} p_i = 1$, x_k denote the inventory level at the beginning of period k, and y_k denote the order-up-to level. It should be noted that y is a decision variable and $y_k \geq x_k$ for $k = 1,\ldots,N$.

The objective is to minimize the total cost, $TC(x)$, over the planning horizon of N periods, which can be expressed as

$$TC(x_1) = E\{\sum_{k=1}^{N}[c(y_k - x_k) + G(y_k) + \delta(y_k - x_k)(L_i + K) - L_i] - cx_{N+1}\} \qquad (1)$$

where $\delta(x)$ is defined as

$$\delta(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \end{cases}$$

For simplification, we assume that at the end of planning horizon N, the unmet demand (or leftover stock) can be produced (or salvaged) at c. This assumption is innocuous since it can be easily relaxed.

By inventory dynamics,

$$x_{k+1} = y_k - \xi_k,\ k = 1,\dots,N \tag{2}$$

and with the assumption about the independent and identically distributed demands, (1) can be readily simplified to

$$TC(x_1) = E\{\sum_{k=1}^{N}[G(y_k) + \delta(y_k - x_k)(L_i + K) - L_i]\} - cx_1 + cN\mu \tag{3}$$

For all x, define $f_{N+1}(x,i) \equiv 0$. Then the dynamic programming equation is

$$f_k(x_k,i) = \min_{y \geq x_k}\{G(y) + \delta(y - x_k)(L_i + K) - L_i + \sum_{j=0}^{M} p_j \int_0^\infty f_{k+1}(y - x, j)d\Phi(x)\} \tag{4}$$

for $k = 1,\dots,N$.

Without loss of generality, we may assume that

$$L_1 \leq L_2 \leq \dots \leq L_M \tag{5}$$

Consequently, the minimum of the total cost can be expressed as

$$TC(x_1) = E[f_1(x_1,i)] - cx_1 + cN\mu \tag{6}$$

3 Optimal Production-inventory Policy

In this section, we characterize the optimal production-inventory policy by using dynamic programming. The analysis consists of two parts: first, we deal with a single period problem and identify the optimal policy for for period N, i.e., the last period in the planning horizon; second, given the optimal policy for the last period, we characterize the optimal policy for the N-period problem through induction.

3.1 Single Period Analysis

For period N in the planning horizon, since $G(y)$ is convex and coercive, there exist a global minimizer of $G(y)$, denoted by S_N, and a solver of $G(y) = G(S_N) + K + L_i$, denoted by $s_N^{(i)}$. In addition, from the convexity of $G(y)$ and $L_i \leq L_{i+1}$ with $i = 1,\dots,M-1$, it can be readily verified that $s_N^{(i)} \geq s_N^{(i+1)}$.

Therefore, the optimal policy is defined by a pair of critical numbers $(s_N^{(i)}, S_N)$. In other words, for $i = 0,1,\dots,M$

$$f_N(x_N,i) = \begin{cases} G(x_N) - L_i, & x_N \geq s_N^{(i)} \\ G(S_N) + K, & x_N < s_N^{(i)} \end{cases} \tag{7}$$

In particular, for the non-peak state, since $L_0 = 0$, we have

$$f_N(x_N,0) = \begin{cases} G(x_N), & x_N \geq s_N^{(0)} \\ G(S_N) + K, & x_N < s_N^{(0)} \end{cases} \tag{8}$$

The optimal policy for non-peak period in our model is still of an (s,S) type, which is different from the base-stock policy conducted in the [Chen et al 2007]'s model without considering setup cost.

3.2 Multi-Period Analysis

This part extends the result of period N to the multi-period problem by induction from dynamic programming. In order to achieve the objective, two lemmas concerning the properties of K-convex function are necessary.

Lemma 1. *(i). If $f(x)$ is K-convex, then it is M-convex for any $M \geq K$. In particular, if $f(x)$ is convex, then it is also K-convex for any $K \geq 0$; (ii). If f and g are K-convex and M-convex, respectively, then $\alpha f + \beta g$ is $(\alpha K + \beta M)$-convex when α and β are positive; (iii). If $f(x)$ is K-convex and a is a random variable such that $E|f(x-a)| < +\infty$ for all x, then $E[f(x-a)]$ is also K-convex; (iv). If $f(x)$ is K-convex, then $f(x)+A$ is also K-convex for any constant $A \in R$.*

Lemma 2. *If $f(x)$ is K-convex and continuous with $f(x) < \infty$ for any finite-valued x and $\lim_{|x|\to\infty} f(x) = \infty$, there exist a value S and a function $g(x)$ such that for any $a \in R$*

$$g(x) = \begin{cases} f(S), & a \leq S \\ \inf_{x \geq a}\{f(x)\}, & a > S \end{cases} \tag{9}$$

Furthermore, $g(x)$ is also K-convex and continuous in x.

In **Lemma 1**, proofs of *(i)* to *(iii)* are given in [Bensoussan et al 1983, Bertsekas 1978], and the proof of *(iv)* is obvious. **Lemma 2** is given in [Chen et al 2007].

Theorem 3. *For any period k, $k = 1,\ldots,N$, there exist pairs of critical numbers $s_k^{(i)}$ and S_k with $s_k^{(i+1)} \leq s_k^{(i)} \leq S_k$, $i = 1,\ldots,M-1$, such that the optimal production-inventory policy is of an $(s_k^{(i)}, S_k)$ type as follows: If $x_k \geq s_k^{(i)}$, take the offer and stop production, and if $x_k < s_k^{(i)}$, reject the offer and produce $(S_k - x_k)$ to the level S_k.*

Proof. We shall show inductively that each of the functions $f_1(x_1,i), f_2(x_2,i),\ldots,f_N(x_N,i)$ is $(K+L_i)$-convex. From the single period analysis in the previous subsection, the result holds for period N. Since S_N is a global minimizer of $G(y)$, and $s_N^{(i)}$ is the solver to the following equation

$$G(y) = G(S_N) + K + L_i,\ y \leq S_N,\ i = 0,1,\ldots,M \tag{10}$$

it follows that $f_N(x_N,i)$ is $(K+L_i)$-convex in x.

Now, we consider the situation of period $N-1$. From (4) and the $f_N(x,i)$, let

$$F_{N-1}(x) = \sum_{i=0}^{M} p_i f_N(x_N,i) \tag{11}$$

We have that for $i = 0,1,\ldots,M$

$$f_{N-1}(x_{N-1},i) = \min_{y \geq x_{N-1}} \{G(y) + \delta(y - x_{N-1})(L_i + K) - L_i + E[F_{N-1}(y - \xi_{N-1})]\} \tag{12}$$

From the results of previous subsection, we know that $f_N(x_N,i)$ is $(K+L_i)$-convex. Then, by **Lemma 1**,

$$G(y) + E[F_{N-1}(y - \xi_{N-1})] \tag{13}$$

is $(K+L_i)$-convex. Thus, by **Lemma 2**, there exist pairs of numbers $s_{N-1}^{(i)}$ and S_{N-1} with $s_{N-1}^{(i)} \leq S_{N-1}$ such that

$$\inf_{y\in(-\infty,+\infty)}\{G(y)+E[F_{N-1}(y-\xi_{N-1})]\} = G(S_{N-1})+E[F_{N-1}(S_{N-1}-\xi_{N-1})] \quad (14)$$

where S_{N-1} is a global minimizer of (13), which performs the same function as S in **Lemma 2**.

In addition, $s_{N-1}^{(i)}$ is the solver to the following equation

$$G(y)+E[F_{N-1}(y-\xi_{N-1})]\} = K+L_i+G(S_{N-1})+E[F_{N-1}(S_{N-1}-\xi_{N-1})],\ y \leq S_N \quad (15)$$

Furthermore,

$$G(y)+E[F_{N-1}(y-\xi_{N-1})] \quad (16)$$

is non-increasing on $(-\infty, s_{N-1}^{(i)}]$; see [Gallego and Sethi 2005]. Consequently, we have

$$\begin{aligned} G(x)+E[F_{N-1}(x-\xi_{N-1})] &\leq G(y)+E[F_{N-1}(y-\xi_{N-1})] \\ &\leq K+L_i+G(y)+E[F_{N-1}(y-\xi_{N-1})] \end{aligned} \quad (17)$$

for any x and y with $s_{N-1}^{i} \leq x \leq y$. Therefore,

$$\begin{aligned} &\min_{y\geq x_{N-1}}\{G(y)+\delta(y-x_{N-1})(L_i+K)-L_i+E[F_{N-1}(y-\xi_{N-1})]\} \\ = &\begin{cases} G(x_{N-1})+E[F_{N-1}(x_{N-1}-\xi_{N-1})]-L_i, & x_{N-1} \geq s_{N-1}^{(i)} \\ G(S_{N-1})+E[F_{N-1}(S_{N-1}-\xi_{N-1})]+K, & x_{N-1} < s_{N-1}^{(i)} \end{cases} \end{aligned} \quad (18)$$

The result holds for period $N-1$.

Now, we define

$$f_{N-1}(x_{N-1},i) = \begin{cases} G(x_{N-1})+E[F_{N-1}(x_{N-1}-\xi_{N-1})]-L_i, & x_{N-1} \geq s_{N-1}^{(i)} \\ G(S_{N-1})+E[F_{N-1}(S_{N-1}-\xi_{N-1})]+K, & x_{N-1} < s_{N-1}^{(i)} \end{cases} \quad (19)$$

By the same line of reasoning for period N together with **Lemma 1**, we conclude that $f_{N-1}(x_{N-1},i)$ is $(K+L_i)$-convex. Moreover, in the proof of period $N-1$, we only use the $(K+L_i)$-convexity property of $f_N(x_N,i)$, thus the same induction procedure can be extended to any period k, $k=1,\ldots,N-2$, with the $(K+L_i)$-convexity of $f_k(x_k,i)$ as the sufficient condition for optimal policies.

The proof is completed. □

The optimal policies characterized in **Theorem 3** can be readily illustrated in Figure 1. Within certain period k, $(s_k^{(0)}, S_k)$ represents the optimal policy for the non-peak state in the left sub-figure, and $(s_k^{(i)}, S_k)$ represents the optimal policy for the i-type peak state in the right sub-figure.

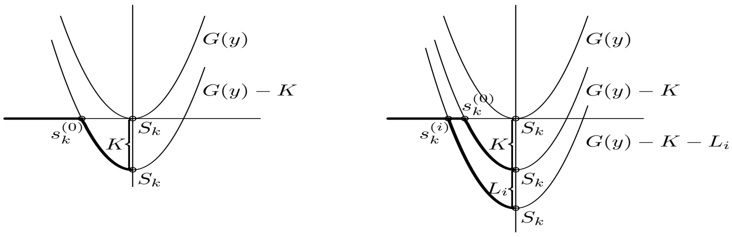

Figure 1: Optimal Policies for Non-Peak and Peak States

4 Conclusion

Based on the model discussed in [Chen et al 2007], this paper proposes a modified model by taking into the consideration of both setup cost and financial compensation to manufacturers for not using energy when the buy-back program is activated during peak states. If the manufacturer stops production and reduces the use of energy, he will be rewarded with a financial compensation associated with different peak state; whereas if he rejects the offer, that is, he decides to continue production without reducing the use of energy, a certain amount of setup cost will incur and no compensation is rewarded.

Through induction, this paper has identified the optimal production-inventory policy as an (s,S) type for all market scenarios. Nevertheless, S_k remains the same whereas $s_k^{(i)}$ varies for different market scenarios. The modified model has shown that within each period k, the relationship among the reproduction levels for different states should satisfy

$$s_k^{(0)} \geq s_k^{(1)} \geq s_k^{(2)} \geq \ldots \geq s_k^{(M)}$$

For each period k, if the inventory level is below $s_k^{(i)}$ for state i, the manufacturer will choose to produce so that the inventory level rises up to S_k; otherwise, he will accept the offer and stop production.

Acknowledgements

This work is partially supported by National Natural Science Foundation of China under Grant #70771013, Programs for NCET-07-0105, Project 211 and the Fundamental Research Funds for the Central Universities.

References

[Bensoussan et al 1983] A. Bensoussan, M. Crouhy, and J.M. Proth, 1983, *Mathematical Theory of Production Planning*, New York: North-Holland.

[Bertsekas 1978] D. Bertsekas, 1978, *Dynamic Programming and Stochastic Control: The Discrete Time Case*, New York: Academic.

[Beyer et al 2006] D. Beyer, F. Cheng, S.P. Sethi, and M.I. Taksar, 2006, *Markovian Demand Inventory Models*, New York: Springer.

[Chao and Chen 2005] X.L. Chao and F.Y. Chen, 2005, Optimal Production and Shutdown Strategy when Supplier Offers Incentives, *Manufacturing & Service Operations Management*, Vol. 7, pp. 130–143.

[Chen et al 2007] Y. Chen, S.P. Sethi, and H. Zhang, 2007, A Production–Inventory Problem for an Energy Buy-Back Program, *IEEE Transaction on Automation Science and Engineering*, Vol. 4, No. 3, pp. 395–406.

[Coy 1999] P. Coy, 1999, Exploiting Uncertainty: The "Real Options" Revolution in Decision Making, *Business Week Online*, June 7, pp. 118–124.

[Gallego and Sethi 2005] G. Gallego, and S.P. Sethi, 2005, $\mathscr{K}$-Convexity in $\Re^n$, *Journal of Optimization Theory and Applications*, Vol. 127, No. 1, pp. 71–88.

[Sethi and Cheng 1997] S.P. Sethi, and F. Cheng, 1997, Optimality of (s,S) Policies in Inventory Models with Markovian Demands, *Operations Research*, Vol. 45, pp. 931–939.

[Song and Zipkin 1993] J.S. Song and P. Zipkin, 1993, Inventory Control in a Fluctuating Demand Environment, *Operations Research*, Vol. 41, pp. 351–370.

[Wald 2000] M.L. Wald, 2000, Utilities Try New Ways to Vary Energy Pricing, *NY Times*, July 17.