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Limited Resource Allocation Games With Activation Cost*

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Abstract In this paper, we investigate resource allocation games with machine activation cost when the resources are limited. We take job scheduling for example. Here, we typically have a fixed number m of identical machines. No machines are initially activated and each machine activated incurs a fixed machine activation cost. Jobs are self-interested players who choose machines with the objective of minimizing their individual cost which is the load on its chosen machine plus its share in the resource's activation cost. The social cost is the sum of the makespan and activation cost of machines. We assess the quality of pure Nash equilibria in terms of the price of anarchy(PoA). First, we present the PoA is less than m when there are m identical machines and provide an instance to prove the bound is tight. Then, we prove that the PoA can be less than $\frac{3}{2}$ and 2 for two special cases of $m = 2$ and $m = 3$ in practical situations, respectively. Finally, we also provide instances to prove the bounds are tight.

Keywords resource allocation game; activation cost ; price of anarchy; cost sharing; Nash equilibrium

1 Introduction

In resource allocation applications, such as job scheduling, telecommunication networks and transportation systems, tasks are assigned to resources to be processed. The Operations Research literature has traditionally treated these problems as combinatorial optimization problems. In the last decade, many of the Operations Research problems have been studied taking into account game theoretic considerations [5],[6]. The terminology of job scheduling is used to present resource allocation games. Jobs are game players. They choose a machine instead of being assigned to a machine by a central designer and act to minimize their own cost rather than optimizing the social objective. The machine which some job chooses to be processed on is the player's strategy. Because of motivated by individual interest, this will usually result in a Nash equilibrium (NE) at which no individual player will benefit from any unilateral deviation for the current resource allocation. This model is recently introduced and studied in [1] and [2]. As shown in [1], this model admits a pure Nash equilibrium which can be computed efficiently when

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available resources are limited. The quality of pure Nash equilibria is concerned because such an equilibrium can often be far from optimal. The notions of the *price of anarchy* (PoA) and the *price of stability* (PoS) are used to analyze the quality of NE solutions. These parameters are introduced by Koutsoupias and Papadimitriou in [10]. The PoA is described as the ratio of the social cost of the worst NE solution and the corresponding optimal social cost.

Since individual cost function decides players' decision making, the structure of this cost function is the heart of any resource allocation problem. The literatures are mainly divided into two main aspects with respect to the individual cost function. Some models emphasize the negative congestion effect, and an individual cost incurred by using a resource is a non-decreasing function of its load ([3],[4],[5],[6]). However, positive congestion effect also happens, and the individual cost is a non-increasing function of its load. Some models assume that each resource has some activation cost, which should be covered by its users. In this case, a machine user wishes to share the resource with additional users as many as possible to minimize his individual cost of using the machine([7],[8],[9]). Recently, Feldman and Tamir [1] propose the model which take both congestion effects into account. With an egalitarian social objective (l_∞ metric), they show that the PoA is unbounded but the PoS is bounded by a tight bound of $\frac{5}{4}$. Bo Chen and Sinan G*ü*rel[2] use the utilitarian social objective (l_1 metric) to assess the NE assignments. They provide a tight parametric bound on the PoA, which is unbounded in general (as in the case of egalitarian social objective considered in [1]).

In this paper, we also consider resource allocation games with the individual cost function which consider both congestion effects as shown before. But, the social cost function we considered is both egalitarian (l_∞ metric) and utilitarian (l_1 metric). In our model, there are limited number of identical machines available, but usage of each machine comes with an additional set-up or activation cost, which is shared in proportion to the job lengths by all jobs assigned to the machine. Such a social objective might be of interest in a situation where system designers wish to trade off their concerns with both the balance of different machine usages and the total number of machines activated. We provide a tight bound on the PoA when the number of machines is m. Furthermore, we prove that the PoA can be less than $\frac{3}{2}$ and 2 for two cases of $m = 2$ and $m = 3$, respectively, and both bounds are tight.

2 Model description and preliminaries

We present our resource allocation games using the terminology of job scheduling for simplicity of presentation.

A job scheduling setting with limited identical machines is characterized by a set of machines $M = \{M_1, M_2, \cdots, M_m\}$, $m \geq 2$, a set of jobs $J = \{1, 2, \cdots, n\}$, $n > 2$, where job $j \in J$ has a length (i.e., processing time) of p_j. W is the total length of all jobs. Let B denote the cost of activating each machine. For notational convenience and without loss of generality, we will assume $B = 1$ in the remainder of our paper. This can be achieved by dividing all job lengths with the activation cost B. It is convenient to divide the jobs into two categories, large and small: $J_l = \{j \in J : p_j > 1\}$ and $J_s = \{j \in J : p_j \leq 1\}$. A schedule is a vector $s = (s_1, s_2, \cdots, s_n)$ which shows that the ith job is processed on $s_i \in M$. Given an overall job schedule s, denote $L_i^s = \sum\limits_{s_j = M_i} p_j$ as the load of machine M_i, then the individual cost of job j is as follows:

$$IC(j) = L_i^s + \frac{p_j}{L_i^s},$$

where the second part which is in proportion to its length with respect to the load of machine M_i represents its share of the activation cost. The social cost function of the overall schedule s is as follows:

$$SC(s) = L_{\max}^s + m_s,$$

where $L_{\max}^s$ and m_s are respective the maximum machine load and the number of machines activated under schedule s.

Let S denote the set of all schedules for problem instance (J,M). $OPT = \min_{s\in S} SC(s)$ is the optimal social cost. Let G be the game we considered. A schedule $s \in S$ is a *pure Nash Equilibrium* (NE) if no player $j \in J$ can benefit from unilaterally deviating from his machine to another machine. Let $\phi(G)$ denote all Nash equilibrium schedules. m^* and m_{NE} are respective the number of machines activated in the optimal schedule and a NE schedule. When schedule s is a NE schedule, then define $L_i^s = L_i$, $L_{\max}^s = L_{\max}$. The number of jobs processed on M_i is defined as n_i.

Definition 2.1. If $\phi(G) \neq \emptyset$:
the price of anarchy (PoA) is the ratio between the social cost of the highest-cost Nash equilibrium schedule and the social optimum, i.e.,

$$PoA = \frac{\max_{s\in\phi(G)} SC(s)}{OPT}.$$

Lemma 2.2. (Michal Feldman, Tami Tamir 2008) For every m, any resource allocation game with m resources induced by the proportional sharing rule admits a pure NE.
Lemma 2.3. (Bo Chen, Sinan *Gürel* 2010) If there are unlimited number of machines available, then any large job will be assigned to a dedicated machine in any NE assignment; for machines of small jobs, $L_i \leq 1$ holds in any NE assignment.
Lemma 2.4. If there are only limited number of machines available, then the difference in load between the most-loaded machine which there is at least one small job processed on and any other machine is no more than 1.

3 The PoA for general case

In this section, we discuss the case of m available machines.
Theorem 3.1. *PoA* $< m$ when there are m machines available, and the bound is tight.

Proof. Given any NE assignment s, we have

$$\begin{aligned}
\frac{SC(s)}{OPT} &\leq \frac{m+W}{\min\{1+W, 2+\frac{W}{2}, \cdots, m+\frac{W}{m}\}} \\
&= \max\{\frac{m+W}{1+W}, \frac{m+W}{2+\frac{W}{2}}, \cdots, \frac{m+W}{m+\frac{W}{m}}\} \\
&= \max_{1\leq i\leq m}\{\frac{m+W}{i+\frac{W}{i}}\} \\
&< \max_{1\leq i\leq m}\{\frac{im+\frac{m}{i}W}{i+\frac{W}{i}}\} \\
&= m.
\end{aligned}$$

Since the above inequality holds for any instance, the upper bound in the theorem is established. The tightness of the upper bound is shown in the following example.

Example 3.2. Consider an instance of mk jobs and m machines, each having a length of $\frac{\varepsilon}{k}$, where $\varepsilon > 0$ and $k > \frac{1}{\varepsilon} - 1$. A NE assignment s is that there are respective k jobs on each machine. Therefore, we conclude that

$$\frac{SC(s)}{OPT} = \frac{m+\varepsilon}{1+m\varepsilon} \to m \quad (\varepsilon \to 0),$$

which equals the upper bound in Theorem 3.1.

4 The PoA for the case of $m = 2$

In this section, we discuss the case of $m = 2$. From Section 2, it is obvious that $PoA < 2$. If the total jobs length is smaller than the activation cost, the problem is indeed trivial. Therefore, we show the PoA is smaller when $W > 1$.

Theorem 4.1. In our considered resource allocation game, $PoA < \frac{3}{2}$ when $m = 2$ and $W > 1$.

Proof. We prove the upper bound of PoA by distinguishing three cases according to the categories of jobs in J.

Case 1. All jobs in J are large jobs, i.e., $p_j > 1, \forall j = 1, 2, \cdots, n$.

Obviously, $m^* = 2, m_{NE} = 2$. Without loss of generality, we assume $L_1 \leq L_2$. Denote $L_2 = L_1 + \sigma, \sigma \geq 0$.

Two subcases are considered according to the number of jobs processed on M_2 in a NE assignment s.

Case 1.1. $n_2 = 1$. Then $SC(s) = OPT$.

Case 1.2. $n_2 \geq 2$

If $0 \leq \sigma \leq 2$, we have

$$\frac{SC(s)}{OPT} \leq \frac{2+L_1+\sigma}{2+L_1+\frac{\sigma}{2}} = 1 + \frac{\frac{\sigma}{2}}{2+L_1+\frac{\sigma}{2}} \leq 1 + \frac{1}{3+\frac{\sigma}{2}} < \frac{4}{3}.$$

If $\sigma > 2$, then we have $\sigma - 2 \leq L_1$. Suppose to the contrary that $\sigma - 2 > L_1$, then $L_2 = L_1 + \sigma > 2L_1 + 2$. If the number of jobs whose lengths $p_j > L_1 + 1$ and $s_j = M_2$ is large than 1, one of these jobs deviates from M_2 to M_1, and we have $IC^{'}(j) \leq L_1 + p_j + 1$, but $IC(j) > L_1 + p_j + 1$, which contradicts s is a NE assignment. So the number of jobs whose length $p_j > L_1 + 1$ and $s_j = M_2$ is no more than 1. Therefore, there must be some job j of $p_j \leq L_1 + 1$. Moving job j from M_2 to M_1, this will result in a reduced individual cost $IC^{'}(j)$:

$$IC^{'}(j) \leq 2L_1 + 2, IC(j) > 2L_1 + 2,$$

which contradicts that s is a NE assignment.

Therefore, we have

$$\frac{SC(s)}{OPT} \leq \frac{2 + L_1 + \sigma}{2 + L_1 + \frac{\sigma}{2}} = 1 + \frac{\frac{\sigma}{2}}{2 + L_1 + \frac{\sigma}{2}} \leq 1 + \frac{\frac{\sigma}{2}}{2 + \sigma - 2 + \frac{\sigma}{2}} = \frac{4}{3}.$$

Case 2. All jobs in J are small jobs, i.e., $p_j \leq 1, \forall j = 1, 2, \cdots, n$.

Two subcases are considered according to the total processing length W.

Case 2.1. $1 < W < 2$, then $2 + \frac{W}{2} > 1 + W$. We can conclude $m^* = 1, m_{NE} = 2$. Then

$$\frac{SC(s)}{OPT} \leq \frac{2 + L_1 + \sigma}{1 + 2L_1 + \sigma} = 1 + \frac{1 - L_1}{1 + 2L_1 + \sigma} < 1 + \frac{1}{2} = \frac{3}{2}.$$

Case 2.2. $W \geq 2$, we can get $m_{NE} = 2$ from Lemma 2.3.

If $m^* = 1$, then

$$\frac{SC(s)}{OPT} \leq \frac{2 + L_1 + \sigma}{1 + 2L_1 + \sigma} = 1 + \frac{1 - L_1}{1 + 2L_1 + \sigma} < 1 + \frac{1}{3} = \frac{4}{3}.$$

If $m^* = 2$, by Lemma 2.4., we have $\sigma \leq 1$. Then

$$\frac{SC(s)}{OPT} \leq \frac{2 + L_1 + \sigma}{2 + L_1 + \frac{\sigma}{2}} = 1 + \frac{\sigma}{2L_1 + \sigma + 4} \leq \frac{7}{6}.$$

Case 3. There are not only large jobs but also small jobs in J.

We also consider two subcases according to W.

Case 3.1. $1 < W < 2$, we get $m^* = 1, m_{NE} = 2$. Then $\frac{SC(s)}{OPT} < \frac{3}{2}$.

Case 3.2. $W \geq 2$, then

(1) $m^* = 1$, then $\frac{SC(s)}{OPT} < \frac{4}{3}$.

(2) $m^* = 2$, if there exist small jobs processed on M_2, we can get $\sigma \leq 1$ by Lemma 2.4. Then

$$\frac{SC(s)}{OPT} \leq \frac{2 + L_1 + \sigma}{2 + L_1 + \frac{\sigma}{2}} = 1 + \frac{\sigma}{2L_1 + \sigma + 4} \leq \frac{7}{6}.$$

if all jobs processed on M_2 are large jobs, we analyze similarly to Case 1. Then

$$\frac{SC(s)}{OPT} \leq \frac{2 + L_1 + \sigma}{2 + L_1 + \frac{\sigma}{2}} = 1 + \frac{\frac{\sigma}{2}}{2 + L_1 + \frac{\sigma}{2}} \leq \frac{4}{3}.$$

Therefore, the upper bound in the theorem is established. The tightness of the bound is shown in the following instance.

Example 4.2. Consider an instance of three jobs, $p_1 = \varepsilon$, $p_2 = \varepsilon$ and $p_3 = 1-\varepsilon$, where $\varepsilon > 0$. A NE assignment s is that job 3 is on M_2, job 1 and job 2 are on M_1. Then

$$\frac{SC(s)}{OPT} = \frac{2+1-\varepsilon}{1+1+\varepsilon} \to \frac{3}{2}(\varepsilon \to 0),$$

which equals the upper bound Theorem 4.1.

5 The PoA for the case of m=3

In this section, we discuss the case of $m = 3$. From Section 2, it is obvious that $PoA < 3$. We show the PoA is smaller when $W > 1$.

Lemma 5.1. In our resource allocation game, $PoA < 2$ when $m = 3$ and $W > 1$.

Proof. We prove the upper bound of PoA by distinguishing three cases according to the categories of jobs in J.

Case 1. All jobs in J are large jobs, i.e., $p_j > 1, \forall j = 1,2,\cdots,n$.

Then we get $m_{NE} = 3$ by Lemma 2.3. Three subcases are considered with respect to the relationships of machine loads in a NE assignment s. Without loss of generality, we can assume $L_1 \le L_2 \le L_3$.

Case 1.1. $L_1 + L_2 \ge L_3 + 2$, then

$$\frac{SC(s)}{OPT} \le \frac{3+L_3}{\min\{2+\frac{L_1+L_2+L_3}{2}, 3+\frac{L_1+L_2+L_3}{3}\}} < \frac{3}{2}.$$

Case 1.2. $L_1 + L_2 \le L_3 + 2$ when $L_3 > 2$. Then $L_3 \ge L_1 + L_2 + 2 > 4$.

Next, we discuss according to the number of jobs processed on M_3 in s.

(1) $n_3 = 1$, then $m^* = 2$, so

$$\frac{SC(s)}{OPT} = \frac{3+L_3}{2+L_3} = 1 + \frac{1}{2+L_3} \le \frac{7}{6}.$$

(2) $n_3 \ge 2$, let $L_3 = L_1 + L_2 + \sigma$, we can conclude $\sigma - 2 \le L_1 + L_2$.

Suppose to the contrary that $\sigma - 2 > L_1 + L_2$, then $L_3 = L_1 + L_2 + \sigma > 2(L_1 + L_2) + 2$. If the number of jobs whose lengths $p_j > L_1 + L_2 + 1$ and $s_j = M_3$ is large than 1, one of these jobs deviates from M_3 to M_1, and we have $IC^{'}(j) \le L_1 + p_j + 1$, but $IC(j) > L_1 + L_2 + p_j + 1$, which contradicts s is a NE assignment. So the number of jobs whose length $p_j > L_1 + L_2 + 1$ and $s_j = M_3$ is no more than 1. Therefore, there must exist job of $p_j \le L_1 + L_2 + 1$. Moving job j from M_3 to M_1, this will result in a reduced individual cost $IC^{'}(j)$:

$$IC^{'}(j) \le 2L_1 + L_2 + 2, IC(j) > 2(L_1 + L_2) + 2,$$

which contradicts s is a NE assignment.

Since $L_1 + L_2 \ge \frac{L_3}{2} - 1$,

$$\frac{SC(s)}{OPT} \le \frac{3+L_3}{\min\{2+\frac{L_1+L_2+L_3}{2}, 3+\frac{L_1+L_2+L_3}{3}\}} < 2.$$

Case 1.3. $L_3 - 2 < L_1 + L_2 < L_3 + 2$.
If $L_3 > 2$,

$$\frac{SC(s)}{OPT} \leq \frac{3+L_3}{\min\{2+\frac{L_1+L_2+L_3}{2}, 3+\frac{L_1+L_2+L_3}{3}\}} < \max\{\frac{3+L_3}{1+L_3},, \frac{3}{2} - \frac{3}{4L_3+14}\} < \frac{3}{2}.$$

If $L_3 \leq 2$, then $L_1 + L_2 \geq L_3, L_3 \geq 1$. So

$$\frac{SC(s)}{OPT} \leq \frac{3+L_3}{\min\{2+\frac{L_1+L_2+L_3}{2}, 3+\frac{L_1+L_2+L_3}{3}\}} \leq \max\{\frac{3+L_3}{2+L_3}, \frac{3}{2} - \frac{9}{4L_3+18}\} < \frac{3}{2}.$$

Case 2. All jobs in J are small jobs, i.e., $p_j \leq 1, \forall j = 1, 2, \cdots, n$.

Three subcases are considered according to the number of machines activated in s.
Case 2.1. $m_{NE} = 1$, then we get $W < 2$. Therefore, $m^* = 1$. We can conclude that $SC(s) = OPT$.
Case 2.2. $m_{NE} = 2$, then we get $L_1 \leq 1, L_2 \leq 1$. Therefore, $W \leq 2$,

$$\frac{SC(s)}{OPT} \leq \frac{2+L_2}{\min\{1+L_1+L_2, 2+\frac{L_1+L_2}{2}\}} < \max\{2, \frac{6}{5}\} = 2.$$

Case 2.3. $m_{NE} = 3$. If $L_3 \leq 1$, because of $L_1 + L_2 + L_3 > 1$, we get

$$\frac{SC(s)}{OPT} \leq \frac{3+L_3}{\min\{1+L_1+L_2+L_3, 2+\frac{L_1+L_2+L_3}{2}, 3+\frac{L_1+L_2+L_3}{3}\}} < 2.$$

If $L_3 > 1$, we get $L_1 + 1 \geq L_3, L_2 + 1 \geq L_3$ from Lemma 2.4., then

$$\begin{aligned}\frac{SC(s)}{OPT} &\leq \frac{3+L_3}{\min\{1+L_1+L_2+L_3, 2+\frac{L_1+L_2+L_3}{2}, 3+\frac{L_1+L_2+L_3}{3}\}} \\ &< \max\{\frac{3+L_3}{3L_3-1}, \frac{3+L_3}{2+\frac{3L_3-2}{2}}, \frac{3+L_3}{3+\frac{3L_3-2}{3}}\} \\ &< \max\{2, \frac{24}{15}, \frac{6}{5}\} \\ &= 2.\end{aligned}$$

Case 3. There are not only large jobs but also small jobs in J.

Obviously, $m_{NE} = 2$ or 3. Two subcases are considered in the following.
Case 3.1. $m_{NE} = 2$. We can conclude there is only one job in J which processed on a dedicated machine and the total length of small jobs is no larger than 1. So

$$\frac{SC(s)}{OPT} \leq \frac{2+L_2}{\min\{1+L_1+L_2, 2+\frac{L_1+L_2}{2}\}} < \max\{\frac{3}{2}, 2\} = 2.$$

Case 3.2. $m_{NE} = 3$.

(1) Small jobs are processed on all machines activated in s. Then $L_3 \geq 1$,$L_1 + 1 \geq L_3$ and $L_2 + 1 \geq L_3$. Similarly to Subcase 2.3., we get

$$\frac{SC(s)}{OPT} \leq \frac{3+L_3}{\min\{1+L_1+L_2+L_3, 2+\frac{L_1+L_2+L_3}{2}, 3+\frac{L_1+L_2+L_3}{3}\}} < 2.$$

(2) There is at least one machine which only processes large jobs among machines activated in s. Under the case, let L denote the load of the most-loaded machine which only processes large jobs , $L^{'}$ and $L^{''}$ denote the other two machines' loads respectively.

(2.1.) $L'+L'' \geq L+2$, then

$$\begin{aligned}\frac{SC(s)}{OPT} &\leq \frac{3+L+1}{\min\{1+L'+L''+L, 2+\frac{L'+L''+L}{2}, 3+\frac{L'+L''+L}{3}\}}\\ &< \max\{\frac{4+L}{2L+3}, \frac{4+L}{L+3}, \frac{4+L}{3+\frac{2L+2}{3}}\}\\ &< \max\{\frac{5}{4}, \frac{3}{2}\}\\ &< 2.\end{aligned}$$

(2.2.) $L'+L^{''} \leq L-2$ when $L>2$. If the number of jobs processed on the machine with load L is 1 and the machine is the most-loaded machine, then

$$\frac{SC(s)}{OPT} \leq \frac{3+L}{\min\{1+L, 2+L, 3+L\}} < \max\{2, \frac{4}{3}, 1\} = 2.$$

If the number of jobs processed on the machine with load L is 1 and the machine is not the most-loaded machine, without loss of generality, we assume L' is the load of the most loaded machine. Then we know there are small jobs processed on it. From $L' > L$ and $L' \leq L''+1$, we also get $L+L'+L'' > L+L+L'-1 > 3L-1$. So

$$\begin{aligned}\frac{SC(s)}{OPT} &\leq \frac{3+L+1}{\min\{1+L'+L''+L, 2+\frac{L'+L''+L}{2}, 3+\frac{L'+L''+L}{3}\}}\\ &< \max\{\frac{4+L}{3L}, \frac{4+L}{2+\frac{3L-1}{2}}, \frac{4+L}{3+\frac{3L-1}{3}}\}\\ &< \max\{\frac{5}{3}, \frac{15}{11}\}\\ &< 2.\end{aligned}$$

If the number of jobs processed on the machine with load L is larger than 1, assume $L = L'+L''+\sigma$. Then we get $\sigma - 2 \leq L'+L''$ using the method shown before. Since $L = L'+L''+\sigma \leq 2(L'+L'')+2$, then $L'+L'' \geq \frac{L}{2}-1$. So

$$\frac{SC(s)}{OPT} \leq \frac{3+L+1}{\min\{1+L'+L''+L, 2+\frac{L'+L''+L}{2}, 3+\frac{L'+L''+L}{3}\}} < 2.$$

(2.3.) $L-2 < L'+L'' < L+2$.
If $L \geq 2$, then

$$\frac{SC(s)}{OPT} \leq \frac{3+L+1}{\min\{1+L'+L''+L, 2+\frac{L'+L''+L}{2}, 3+\frac{L'+L''+L}{3}\}} < \max\{2, \frac{18}{11}\} = 2.$$

If $L \leq 2$, it is to say that there is only one large job processed on the machine of load L. Analyzing similarly to the Case (2.2.), we get $\frac{SC(s)}{OPT} < 2$.

Example 5.2. Consider an instance of $2k+1$ jobs, $2k$ jobs of length $\frac{\varepsilon}{k}$ together with a job of length $1-\varepsilon$ where $k > \frac{1}{\varepsilon} - 1$ and $\varepsilon > 0$. A NE assignment s is that there are respective k jobs on M_1 and M_2, the job of length $1-\varepsilon$ is processed on M_3. So

$$\frac{SC(s)}{OPT} = \frac{3+1-\varepsilon}{1+1+\varepsilon} \to 2 \quad (\varepsilon \to 0).$$

Therefore, we get the upper bound.

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