On the Adjacent Vertex-Distinguishing Equitable-Total Chromatic Number of $P_m \vee F_n$*

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Abstract *In this paper, we obtained the the* adjacent vertex-distinguishing equitable-total chromatic number *of* $P_m \vee F_n$*,where,*$P_m \vee F_n$ *is join-graph of path with order* n *and fan with order* $n+1$.

Keywords graph, adjacent vertex-distinguishing total coloring of graphs, adjacent vertex-distinguishing equitable-total coloring of graphs

1 Introduction

It is a very hard to solving the vertex-distinguishing edge coloring(or strong coloring) of graphs studied in paper [1-5] introduced from the theory of network. It is also hard to solve the adjacent strong edge coloring (or adjacent vertex-distinguishing edge coloring of graphs introduced in paper [6] and adjacent vertex-distinguishing total coloring of graphs introduced in paper [7]. In paper [8-9], the concept that vertex-distinguishing equitable total coloring of graphs and adjacent vertex-distinguishing equitable-total chromatic number of graphs is given to study some graphs. In this paper, we give a method to solve $P_m \vee F_n$. All of the graphs concerned in this paper are simple, finite and undirected graph . We denote by $V(G)$, $E(G)$ and $\Delta(G)$ the set of vertices , edges and the maximum degree of graph G, respectively.

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Definition 1 [7] Let $G(V, E)$ is a connect graph of which the order is at least 2, k is an positive integer and f is the mapping from $V(G) \cup E(G)$ to $\{1, 2, \cdots, k\}$. For any $v \in V(G)$, if

1. for any $uv, vw \in E(G), u \neq w$, there is $f(uv) \neq f(vw)$;
2. for any $uv \in E(G), u \neq v$, there is $f(u) \neq f(v), f(u) \neq f(uv), f(v) \neq f(uv)$;
3. for any $uv \in E(G), u \neq v$, there is $C(u) \neq C(v)$

Where $C(u) = \{f(u)\} \cup \{f(uv)|uv \in E(G)\}$. Then f is called a k-adjacent vertex-distinguishing of coloring of graph G(in brief, denoted by k-AVDTC) and $\chi_{at}(G) = \min\{k|G \quad \text{has} \quad k\text{-AVDTC}\}$ is called the adjacent vertex-distinguishing total chromatic number of graph G.

It is obviously that for any graph $G(|V(G)| \geq 2)$, $\chi_{at}(G)$ exists.

Obviously for graph G, if $uv \in G$ and $d(u) = d(v) = \Delta(G)$, then

$$\chi_{at}(G) \geq \Delta(G) + 2.$$

In paper [7], adjacent vertex-distinguishing total chromatic numbers of some graphs are obtained and a conjecture is given.

Conjecture 1 [7] For graph G,

$$\chi_{at}(G) \leq \Delta(G) + 3.$$

Definition 2 [9] For graph G, let $f : V(G) \cup E(G) \to \{1, 2, \cdots, k\}$ be a k-AVDTC of G. Let $S_i = V_i \cup E_i$. If for any $i, j \in \{1, 2, \cdots, k\}$,

$$||S_i| - |S_j|| \leq 1$$

then f is called a adjacent vertex-distinguishing equitable-total coloring of G(in brief, denoted by k-AVDETC), where

$$V_i = \{u \in V(G)|f(u) = i\}, E_i = uv \in E(G)|f(uv) = i, i = 1, 2, \cdots, k.$$

And

$$\chi_{aet}(G) = \min\{k|G \text{ has a } k\text{-AVDETC of } G\}$$

is called adjacent vertex-distinguishing equitable-total chromatic number of G.

Obviously for graph G, $\chi_{aet}(G) \geq \chi_{at}(G)$.

Conjecture 2 [9] For any graph,then

$$(1)\chi_{aet}(G) \leq \Delta(G) + 3;$$

$$(2)\chi_{aet}(G) = \chi_{at}(G)$$

Definition 3[12] For graph G and $H(V(G) \cap V(H) = E(G) \cap E(H) = \phi)$, a new graph induced by G, H is called graph G join H(denoted by $G \vee H$) if

$$V(G \vee H) = V(G) \cup V(H),\ E(G \vee H) = E(G) \cup E(H) \cup \{uv|u \in V(G), v \in V(H)\}.$$

In the [9] , we have get the adjacent vertex-distinguishing equitable-total chromatic numbers of some graphs such as path, circle, complete graph, complete bipartite graph, fan, wheel, path join path, path join circle, circle join circle, path join star, path join wheel.In this paper we get the adjacent vertex-distinguishing equitable-total chromatic numbers of $P_m \vee F_n$. The other terminologies and mark refer to [10-12].

2 Main Results

Lemma 1 [7].Let K_n be a simple graph with order n, then

$$\chi_{aet}(K_n) = \begin{cases} n+1, & n \equiv 0(mod2); \\ n+2, & n \equiv 1(mod2). \end{cases}$$

Lemma 2 [7].Let G be a simple graphs and $uv \in E(G), d(u) = d(v) = \Delta(G)$, then

$$\chi_{aet}(G) \geq \Delta(G) + 2$$

Suppose P_m is a path with order m, $P_m = u_1u_2 \cdots u_m$;

$$V(F_n) = \{v_i \mid i = 0, 1, \cdots, n\};$$

$$E(F_n) = \{v_0v_i \mid i = 1, 2, \cdots, n\} \cup \{v_iv_{i+1} \mid = 1, 2, \cdots, n-1\}$$

Theorem 1.For $n = 2$, then

$$\chi_{aet}(P_m \vee F_2) = \begin{cases} 5 & m = 1 \\ 7 & m = 2 \\ m+4 & m \geq 3 \end{cases}$$

Proof. Owing to $P_1 \vee F_2 = K_4, P_2 \vee F_2 = K_5$,according to Lemma 1,we know conclusion is true.

When $m \geq 3$ owing to $d(v_0) = d(v_1) = m + 2 = \Delta(P_m \vee F_2)$,according to Lemma 2,we know

$$\chi_{aet}(P_m \vee F_2) \geq m + 4$$

To certify theorem is true, we only give a (m+4)-AVDETC of $P_m \vee F_2, (m \geq 3)$

Let f be :

$$f(v_0v_i) = i, \quad i = 1, 2;$$

$$f(v_0u_i) = 2 + i, \quad i = 1, 2, \cdots, m;$$

$$f(v_1v_2) = 4;$$

$$f(u_iv_j) = i + j + 3(mod(m+4)), \quad i = 1, 2, \cdots, m; j = 1, 2;$$

$$f(v_i) = 1 + i, \quad i = 1, 2;$$

$$f(v_0)=0;$$

$$f(u_i)=i+3,\quad i=1,2,\cdots,m.$$

For n order path $P_n(n\geq 2)$,

$$\chi_{aet}(P_n)=\begin{cases}3, & n=2,\ 3;\\ 4, & n\geq 4.\end{cases}$$

Case 1.When $3\leq m\leq 6$ $f(u_iu_{i+1})=i, i=1,2,\cdots,m-1$..Obviously f is a (m+4)-AVDETC of $P_m\vee F_2,(m\geq 3)$

Case 2.When $7\leq m\leq 10$,then

$$f(u_iu_{i+1})=\begin{cases}i & i=1,2,\cdots,6\\ 7+i(mod(m+4)) & i=6,7,\cdots,m-1\end{cases}$$

Obviously f is a (m+4)-AVDETC of $P_m\vee F_2,(m\geq 3)$

Case 3.When $m\geq 11$,then

$$f(u_iu_{i+1})=\begin{cases}i & i=1,2,\cdots,5\\ 8+i(mod(m+4)) & i=6,7,\cdots,m-1\end{cases}$$

Obviously f is a (m+4)-AVDETC of $P_m\vee F_2,(m\geq 3)$,and

$$|\ S_i\ |=\begin{cases}4 & i=1,2,\cdots,13,m+3,0\\ 5 & i=14,15,\cdots,m+2\end{cases}$$

So f also is a (m+4)-AVDETC of $P_m\vee F_2,(m\geq 3),(m\geq 11)$.

Above all,we know when $m\geq 3$, $P_m\vee F_2$ exists (m+4)-AVDETC,so theorem is true.

Theorem 2. For $n\geq 3$,$m=1,2$, then

$$\chi_{aet}(P_m\vee F_n)=\begin{cases}n+3 & m=1\\ n+4 & m=2\end{cases}$$

Proof. Owing to

$$\Delta(P_m\vee F_n)=\begin{cases}n+1 & m=1\\ n+2 & m=2\end{cases}$$

And $d(v_0)=d(u_1)=\Delta(P_m\vee F_n),(m=1,2)$, and $v_0u_1\in E(P_m\vee F_n)$, so $\chi_{aet}(P_1\vee F_n)\geq m+3$ and $\chi_{aet}(P_2\vee F_n)\geq n+4$ by lemma 2.

When m=1, we only give a (n+3)-AVDETC of $P_1\vee F_n$.

Let f be :

$$f(v_0v_i)=i,\quad i=1,2,\cdots,n;$$

$$f(v_0u_1)=n+1;$$

$$f(v_i)=i+1,\quad i=1,2,\cdots,n;$$

$$f(u_1)=n+2$$

$$f(v_0) = 0;$$

$$f(v_i v_{i+1}) = 3 + i, \quad i = 1, 2, \cdots, n-1;$$

$$f(u_1 v_i) = n + 2 + i(mod(n+3)), \quad i = 1, 2, \cdots, n$$

Obviously f is a $(n+3) - AVDETC$ of $P_1 \vee F_n, (n \geq 3)$.

When m=2,let f be:

$$f(v_0 v_i) = i, \quad i = 1, 2, \cdots, n;$$

$$f(v_0 u_i) = n + i, \quad i = 1, 2;$$

$$f(v_i) = i + 1, \quad i = 1, 2, \cdots, n;$$

$$f(u_i) = n + 1 + i, \quad i = 1, 2$$

$$f(v_0) = 0;$$

$$f(v_i v_{i+1} = 3 + i, \quad i = 1, 2, \cdots, n-1;$$

$$f(u_i v_j) = m + 1 + i + j(mod(n+4)), \quad i = 1, 2; j = 1, 2, \cdots, n.$$

Obviously, f is a (n+4)-AVDETC.

Above all, theorem 2 is true.

Theorem 3 If $m \geq 3, n \geq 3$, then

$$\Delta(P_m \vee F_n) = \begin{cases} m+n+2 & m = 3 \ or \ n = 3 \\ m+n+1 & m \geq n \geq 4 \ or \ n > m \geq 4 \end{cases}$$

Proof. We now consider the following cases separately

Case 1.When n=3, owing to $\Delta(P_m \vee F_3) = m + n$ and $d(v_0) = d(v_2) = m + n, v_0 v_2 \in E(P_m \vee F_3)$,so $\chi_{aet}(P_m \vee F_3) \geq m + n + 2$ by lemma 2.

Let f be:

$$f(v_0 v_i) = i, \quad i = 1, 2, 3;$$

$$f(v_0 u_i) = i + 3, \quad i = 1, 2, \cdots, m;$$

$$f(v_i v_{i+1}) = 3 + i, \quad i = 1, 2;$$

$$f(u_i u_{i+1}) = 1 + i, \quad i = 1, 2, \cdots, m;$$

$$f(v_i) = 1 + i, \quad i = 1, 2, 3;$$

$$f(v_0) = 0;$$

$$f(u_i) = 4 + i, \quad i = 1, 2, \cdots, m;$$

$$f(v_i v_{i+1}) = 3 + i, \quad i = 1, 2.$$

When m=3,

$$f(u_i v_j) = 4 + i(mod 8), i = 1, 2, 3; j = 1, 2, 3; f(u_i u_{i+1}) = i + 1, i = 1, 2;$$

For f, we have

$$\overline{C}(v_0)=\{7\},\overline{C}(v_1)=\{3,5\},\overline{C}(v_2)=\{6\},\overline{C}(v_3)=\{6,7\},$$

$$\overline{C}(u_1)=\{1,3\},\overline{C}(u_2)=\{4\},\overline{C}(u_3)=\{4,5\}.$$

So f is a 8-AVDTC of $P_3 \vee F_3$.And

$$|\ S_i\ |=\begin{cases} 3 & i=1,3,4,5,6,7; \\ 4 & i=0,2 \end{cases}$$

So f is a 9-AVDETC.

When m=5,

$$f(u_iu_{i+1})=i, \quad i=1,2,3,4;$$

$$f(u_iv_j)=4+i+j, \quad i=1,2; j=1,2,3;$$

$$f(u_3v_j)=8+j(mod 10), \quad j=1,2,3;$$

$$f(u_4v_1)=0;$$

$$f(u_4v_2)=9;$$

$$f(u_4v_3)=2;$$

$$f(u_5v_j)=4+j, \quad j=1,2;$$

$$f(u_5v_3)=0.$$

For f,we have

$$\overline{C}(v_0)=\{9\},\overline{C}(v_1)=\{3,8\},\overline{C}(v_2)=\{1\},\overline{C}(v_3)=\{6,7\},\overline{C}(u_5)=\{1,2,3,7\}$$

$$\overline{C}(u_1)=\{2,3,9,0\},\overline{C}(u_2)=\{3,4,0\},\overline{C}(u_3)=\{4,5,8\},$$

$$\overline{C}(u_4)=\{1,5,6\},\overline{C}(v_0)=\{7\}$$

So f is a 10-AVDTC of $P_5 \vee F_3$, and

$$|\ S_i\ |=\begin{cases} 3 & i=1,3; \\ 4 & i=2,4,5,6,7,8,9,0 \end{cases}$$

So f also is a 10-AVDETC of $P_5 \vee F_3$.

When m=6,

$$f(u_iv_j)=5+i+j(mod 11), i=2,3,4,5; j=1,2,3; f(u_1v_j)=5+j, j=1,2,3;$$

$$f(u_6v_1)=3, f(u_6v_2)=6, f(u_6v_3)=7.$$

Obviously f is a 11-AVDETC of $P_6 \vee F_3$.

When $7 \le m \le 8$

$$f(u_1v_j)=5+j, \quad j+1,2,3;$$

$$f(u_iv_j)=5+i+j(mod(m+5)), \quad i=2,3,\cdots,m-1; j=1,2,3$$

$$f(u_m v_1) = 3; f(u_m v_2) = 6; f(u_m v_3) = 7.$$

If m=7,$f(u_i u_{i+1}) = i - 1, i = 1, 2, \cdots, 6$

Obviously f is a 12-AVDETC of $P_7 \vee F_3$.

If $m = 8.f(u_i u_{i+1}) = i, i = 1, 2, \cdots, 6; f(u_7 u_8) = 4$.Obviously f is 11-AVDTC of $P_8 \vee F_3$,and

$$| S_i |= \begin{cases} 5 & i = 4, 6, 10, 11 \\ 4 & others \end{cases}$$

So f is 13-AVDETC of $P_8 \vee F_3$.

If m=9,

$$| S_i |= \begin{cases} 4 & i = 0, 1, 2, 6, 7, 8, 9, 13 \\ 5 & others \end{cases}$$

If m=10

$$| S_i |= \begin{cases} 4 & i = 0, 1, 2, 7, 8, 9, 14 \\ 5 & others \end{cases}$$

If m=11

$$| S_i |= \begin{cases} 4 & i = 0, 1, 2, 8, 9, 15 \\ 5 & others \end{cases}$$

If m=12

$$| S_i |= \begin{cases} 4 & i = 0, 1, 2, 9, 16 \\ 5 & others \end{cases}$$

If m=13

$$| S_i |= \begin{cases} 4 & i = 0, 1, 2, 17 \\ 5 & others \end{cases}$$

So f also is a (m+5)-AVDETC of $P_m \vee F_3, (9 \leq m \leq 13)$

If m=14

$$| S_i |= \begin{cases} 4 & i = 7, 8, 9 \\ 5 & others \end{cases}$$

If m=15

$$| S_i |= \begin{cases} 4 & i = 8, 9 \\ 5 & others \end{cases}$$

If m=16

$$| S_i |= \begin{cases} 4 & i = 9 \\ 5 & others \end{cases}$$

If m=17,$| S_i |= 5, i = 0, 1, \cdots, m + 4$ So f also is (m+5)-AVDETC of $P_m \vee F_3, 14 \leq m \leq 17$

When $m \geq 18$

$$f(u_i u_{i+1}) = m + 3 + i(mod(m + 5)), \quad i = 1, 2, \cdots, m - 6;$$

$$f(u_i u_{i+1}) = i + 6 - m, \quad i = m - 5, m - 4, m - 3, m - 2, m - 1.$$

Obviously f is a (m+5)-AVDTC of $P_m \vee F_3$.and

$$| S_i |= \begin{cases} 6 & i = 10, 11, \cdots, m-8 \\ 5 & others \end{cases}$$

So f is a (m+5)-AVDTEC of $P_m \vee F_3, (m \geq 18)$

Case 2.When $m = 3, n \geq 4$,$\Delta(P_3 \vee F_n) = n+3$ and $d(v_0) = d(u_2) = n+3$,so $\chi_{aet}(P_3 \vee F_n) \geq n+5$ by lemma 2,same as Case 1,(regard P_3 as $v_1v_2v_3$),we can obtain (n+5)-AVDEC of $P_3 \vee F_n, (n \geq 4)$

Case 3.When $m \geq 4$ and $n \geq 4$,$\Delta(P_m \vee F_n) = m+n$ and only $v_0, d(v_0) = m+n$..To certify conclusion is true,we only give a (m+n+1)-AVDETC of $P_m \vee F_n, (m \geq 4, n \geq 4)$

Subcase 3.1 When $n \geq 7$,let f be :

$$f(v_0v_i) = i, \quad i = 1, 2, \cdots, n;$$

$$f(v_0u_i) = n+i, \quad i = 1, 2, \cdots, n;$$

$$f(u_n) = 1; f(v_0) = 0;$$

$$f(v_i) = i+1, \quad i = 1, 2 \cdots, n$$

$$f(u_i) = n+1+i, \quad i = 1, 2, \cdots, n-1;$$

$$f(v_iv_{i+1}) = i-1, \quad i-1, 2, \cdots, n-1;$$

$$f(u_iu_{i+1}) = n-1+i, \quad i = 1, 2, \cdots, n-1$$

$$f(u_1v_i) = n+2+i(mos(2n+1)), \quad i = 1, 2, \cdots, n-1;$$

$$f(u_1v_n) = n-1$$

If $n \equiv 1(mod2)$,

$$f(u_iv_j) = n+1+i+j(mod(2n+1)), i = 2, 3, \cdots, \frac{n+1}{2}; j = 1, 2, \cdots, n$$

$$f(u_iv_j) = 3 - \frac{n+5}{2} + i + j, i = \frac{n+3}{2}, \frac{n+5}{2}, \cdots, n; j = 1, 2, \cdots, n;$$

If $n \equiv 0(mod2)$,

$$f(u_iv_j) = n+1+i+j(mod(2n+1), i = 2, 3, \cdots, \frac{n}{2}+1; j = 1, 2, \cdots, n)$$

$$f(u_iv_j) = i+j-\frac{n}{2}, i = \frac{n}{2}+2, \frac{n}{2}+3, \cdots, n; j = 1, 2, \cdots, n.$$

It is clear f is a (2n+1)-AVDETC of $P_n \vee F_n, (n \geq 7)$.

Subcase 3.2 If $m > n \geq 4$,or $n > m \geq 4$, suppose that $m > n \geq 4$

Subcase 3.2.1 If $m = n+1 \geq 5$,when n=4,let f be:

$$f(v_0v_i) = i, \quad i = 1, 2, 3, 4;$$

$$f(v_0u_i) = 4 + i, \quad i = 1, 2, 3, 4, 5;$$

$$f(v_0) = 0;$$

$$f(v_i) = 1 + i, \quad i = 1, 2, 3, 4;$$

$$f(u_i) = 5 + i, \quad i = 1, 2, 3, 4;$$

$$f(u_5) = 1;$$

$$f(u_iv_j) = 5 + i + j(mod10), \quad i = 1, 2, 3, 4; j = 1, 2, 3, 4;$$

$$f(u_5v_1) = 3, f(u_5v_2) = 6, f(u_5v_3) = 7, f(u_5v_4) = 8;$$

$$f(v_iv_{i+1}) = 3 + i, \quad i = 1, 2, 3;$$

$$f(u_iu_{i+1}) = i + 3, \quad i = 1, 2, 3;$$

$$f(u_4u_5) = 4.$$

For f,we have :

$$\overline{C}(u_1) = \{1, 2, 3\}; \overline{C}(u_2) = \{2, 3\}; \overline{C}(u_3) = \{3, 4\};$$

$$\overline{C}(u_4) = \{5, 7\}; \overline{C}(u_5) = \{2, 5, 0\}; \overline{C}(v_1) = \{6, 5\}; \overline{C}(v_2) = \{7\}$$

$$\overline{C}(v_3) = \{8\}; \overline{C}(v_4) = \{7, 9\}; \overline{C}(v_0) = \phi$$

So f is a 10-AVDTC of $P_5 \vee F_4$.
If $n \geq 5$,

$$f(v_0v_i) = i, \quad i = 1, 2, \cdots, n;$$

$$f(v_0u_i) = n + i, \quad i = 1, 2, \cdots, n + 1;$$

$$f(v_0) = 0,$$

$$f(v_i) = 1 + i, \quad i = 1, 2, \cdots, n.$$

$$f(u_{n+1}) = 1;$$

$$f(u_i) = n + 1 + i, \quad i = 1, 2, \cdots, n;$$

$$f(v_iv_{i+1}) = n + i, \quad i = 1, 2, \cdots, n - 1;$$

$$f(u_iu_{i+1}) = n - 1 + i, \quad i = 1, 2, \cdots, n.$$

If n=5,

$$f(u_iv_j) = 6 + i + j(mod12), i = 1, 2, 3; j = 1, 2, 3, 4, 5; f(u_4v_i) = i - 1, i = 1, 2, 3, 4, 5$$

$$f(u_5v_i) = 2 + i, i = 1, 2, 3, 4, 5; f(u_6v_i) = i + 3, i = 1, 2, 3; f(u_6v_i) = i - 3, i = 4, 5.$$

For f, we have

$$\overline{C}(v_0)=\phi;\overline{C}(v_1)=\{5,7,11\};\overline{C}(v_2)=\{0,8\};\overline{C}(v_3)=\{1,9\}$$

$$\overline{C}(v_4)=\{7,10\};\overline{C}(v_5)=\{8,10,11\};\overline{C}(u_1)=\{1,2,3,4\};\overline{C}(u_2)=\{2,3,4\}$$

$$\overline{C}(u_3)=\{3,4,5\};\overline{C}(u_4)=\{5,6,11\};\overline{C}(u_5)=\{0,1,2\};\overline{C}(u_6)=\{0,7,8,10\}$$

So f is a 12-AVDTC of $P_6 \vee F_5$,and

$$\mid S_i \mid=\begin{cases} 6 & i=9 \\ 5 & others \end{cases}$$

So f is a 12-AVDETC of $P_6 \vee F_5$
If n=6,

$$f(v_iv_{i+1})=6+i,\quad i=1,2,3,4,5;$$
$$f(u_iu_{i+1})=4+i,\quad i=1,2,3,4,5,6;$$
$$f(u_iv_j)=7+i+j(mod14),\quad i=1,2,3,4;j=1,\cdots,6;$$
$$f(u_5v_i)=i-1,\quad i=1,\cdots,6;$$
$$f(u_6v_i)=2+i,\quad i=1,\cdots,6;$$
$$f(u_7v_i)=3+i,\quad i=1,2,3;$$
$$f(u_7v_i)=i-2,\quad i=4,5;$$
$$f(u_7v_6)=9.$$

So f is a 14-AVDTC of $P_7 \vee F_6$, and

$$\mid S_i \mid=\begin{cases} 5 & i=4,6,7,8 \\ 6 & others \end{cases}$$

Same as it,we can obtain when $n \geq 7$, f is a 2(n+1)-AVDETC of $P_{n+1} \vee F_n$.
Subcase 3.3. $m=n+k, k\geq 2, n\geq 4$.It is clear f is a (m+n+1)-AVDETC.
Above all, theorem is true.

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