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The Auxiliary Problem Algorithm for Generalized Linear Complementarity Problem Over a Polyhedral Cone*

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Abstract In this paper, we consider an auxiliary problem algorithm for solving the generalized linear complementarity problem over a polyhedral cone (GLCP). First, we equivalently reformulate the GLCP as an affine variational inequalities problem over a polyhedral cone via a linearly constrained quadratic programming under suitable assumptions, based on which we propose an auxiliary problem method to solve the GLCP and establish its global convergence. A numerical experiments of the method are also reported in this paper.

Keywords GLCP, auxiliary problem method, global convergence

1 Introduction

Let $F(x) = Mx + p, G(x) = Nx + q$, where $M,N \in R^{m\times n},\ p,q \in R^m$. The generalized linear complementarity problem, abbreviated as GLCP, is to find a vector $x^* \in R^n$ such that

$$F(x^*) \in \mathscr{K}, \quad G(x^*) \in \mathscr{K}^0, \quad F(x^*)^\top G(x^*) = 0, \tag{1.1}$$

where $\mathscr{K}$ is a polyhedral cone in R^n, that is, there exists $A \in R^{s\times m}, B \in R^{t\times m}$, such that $\mathscr{K} = \{v \in R^m \mid Av \geq 0,\ Bv = 0\}$. It is easy to verify that its polar cone $\mathscr{K}^\circ$ assumes the following from ([1,2])

$$\mathscr{K}^\circ = \{u \in R^m \mid u = A^\top \lambda_1 + B^\top \lambda_2, \lambda_1 \in R^s_+, \lambda_2 \in R^t\}.$$

Throughout this paper, we denote the "feasible" region of the GLCP by X, i.e.,

$$X = \{x \in R^n \mid A(Mx+p) \geq 0,\ B(Mx+p) = 0,\ Nx+q = A^\top \lambda_1 + B^\top \lambda_2\},$$

and the solution set of the GLCP is denoted by X^* which is always assumed to be nonempty.

The GLCP is a special case of the generalized nonlinear complementarity over a polyhedral cone (GCP) which was firstly considered by Andreani et al. in [1]

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and further developed by Wang et al. in [2]. The GCP plays a significant role in economics, operation research and nonlinear analysis, etc, and has been received much attention of researchers([1,2,3,4]).

In recent years, many effective methods have been proposed for solving GCP. The basic idea of these methods is to reformulate the problem as an unconstrained or simply constrained optimization problem([1,2,3,4]). Different from the algorithms listed above, we adopt the auxiliary problem algorithm for solving the GLCP, based on the algorithm given by Ferris and Mangasarian in [5] to solve the affine monotone variational inequality, but convergence result of it wasn't proved. The main contribution of the paper is two folds: first, we equivalently reformulate the GLCP as an affine variational inequalities problem over a polyhedral cone via a linearly constrained quadratic programming under suitable assumptions, based on which we establish the global convergence of the method, second, Compared with the algorithm converges globally in [2,3], our conditions are weaker. A numerical experiments are also reported.

The following notation will be used throughout the paper. R^n denote the n-dimensional Euclidean space, all vectors are column vectors, $(x^\top, y^\top)^\top$ is represented as (x,y) for convenience, $A^\top$ denotes transposition of matrix A, $\|x\|$ denotes the Euclidean 2-norm of a vector x.

2 The equivalent reformulation of the GLCP

In this section, we will establish an equivalent reformulation of the GLCP. First, we give the needed assumptions.

Assumption 2.1. The matrix $M^\top N$ is positive semi-definite in X (not necessarily symmetric), where M and N are the matrices defined in (1.1).

For problem (1.1), we have the following conclusion.

Lemma 2.1 x^* is a solution of GLCP if and only if there exist $\lambda_1^* \in R^s, \lambda_2^* \in R^t$ such that

$$\begin{cases} AF(x^*) \geq 0, BF(x^*) = 0, \lambda_1^* \geq 0 \\ G(x^*) = A^\top \lambda_1^* + B^\top \lambda_2^*, F(x^*)^\top G(x^*) = 0 \end{cases}$$

For any x in X, i.e., the feasible region of the GLCP, we have

$$(Mx+p)^\top (Nx+q) = (Mx+p)^\top A^\top \lambda_1 + (Mx+p)^\top B^\top \lambda_2 \geq 0.$$

Let $y = (x, \lambda_1, \lambda_2) \in R^{n+s+t}$. From Lemma 2.1 and discussion above, the problem (1.1) can be equivalently reformulated as the following constrained optimization problem

$$\begin{array}{ll} \min & f(y) = (Mx+p)^\top (Nx+q) \\ \text{s.t.} & \begin{pmatrix} AM & 0 & 0 \\ 0 & I & 0 \end{pmatrix} y \geq \begin{pmatrix} -Ap \\ 0 \end{pmatrix}, \\ & \begin{pmatrix} N & -A^\top & -B^\top \\ BM & 0 & 0 \end{pmatrix} y = \begin{pmatrix} -q \\ -Bp \end{pmatrix}. \end{array} \tag{2.1}$$

in the sense that x^* is a solution of (1.1) if and only if there exist $\lambda_1^* \in R_+^s, \lambda_2^* \in R^t$ such that $y^* = (x^*, \lambda_1^*, \lambda_2^*)$ is a global optimal solution of (2.1) with the objective varnishing. Furthermore, the objective function of (2.1) can be rewritten as $f(y) = \frac{1}{2}x^\top \hat{M}x + \hat{q}^\top x + p^\top q$, where $\hat{M} = M^\top N + N^\top M, \hat{q} = M^\top q + N^\top p$. Under Assumption 2.1, it is easy to check that the Hessian matrix of $f(y)$, $\bar{M} = \begin{pmatrix} \hat{M} & 0 \\ 0 & 0 \end{pmatrix}$, is positive semi-definite, so $f(y)$ is a convex function. By the convex optimization theory, we know that the stationary set of (2.1) coincides with its solution set which also coincides with the solution set of the following variational inequality problem: find $y^* \in \bar{\Omega}$ such that

$$(y - y^*)^\top (\bar{M}y^* + \bar{q}) \geq 0, \quad \forall y \in \bar{\Omega}, \tag{2.2}$$

where $\bar{\Omega}$ denotes the domain set of problem (2.1) and $\bar{q} = \begin{pmatrix} \hat{q} \\ 0 \end{pmatrix} \in R^{n+s+t}$.

From the analysis above, we have the following conclusion.

Theorem 2.1 A point $x^* \in R^n$ is a solution of GLCP if and only if there exist $\lambda_1^* \in R_+^s, \lambda_2^* \in R^t$ such that $y^* = (x^*, \lambda_1^*, \lambda_2^*)$ is a solution of (2.2).

3 Auxiliary problem method and Convergence

In [5], Ferris and Mangasarian proposed an auxiliary problem method for solving the affine monotone variational inequality, but convergence result of it wasn't proved. Here, we adopt the method to solve the GLCP and establish its convergence based on the analysis above. Now, we formally state our algorithm.

Algorithm 3.1

Step1. Given $\varepsilon > 0$, $\gamma > \frac{1}{2}\mu_{max}$, where μ_{max} is the maximum eigenvalue of $\bar{M}$. Choose any initial point $y^0 \in R^{n+s+t}$. Set $k \triangleq 0$;

Step2. Compute $y^{k+1} = y(y^k)$ by solving the following quadratic programming

$$\begin{array}{ll} \min & (y - y^k)^\top (\bar{M}y + \bar{q}) + \frac{1}{2}\gamma \|y - y^k\|^2 \\ \text{s.t.} & y \in \bar{\Omega}. \end{array} \tag{3.1}$$

Step3. If $\|y^{k+1} - y^k\| \leq \varepsilon$ stop, otherwise, go to Step 2 with $k \triangleq k+1$.

We are now in the position to show the convergence of the algorithm.

Lemma 3.1 Suppose Assumption 2.1 holds, x^* is a solution of (1.1), and μ_{max} is a maximum eigenvalue of $\bar{M}$, we have

(i) if $\mu_{max} \neq 0$, then $\langle \bar{M}y + \bar{q},\ y - y^* \rangle \geq (1/\mu_{max}) \|\bar{M}(y - y^*)\|^2, \quad \forall y \in \bar{\Omega}$.

(ii) if $\mu_{max} = 0$, then $\langle \bar{M}y + \bar{q},\ y - y^* \rangle \geq 0, \quad \forall y \in \bar{\Omega}$.

Proof. Under Assumption 2.1, there exists an orthogonal matrix P such that

$$P\bar{M}P^\top = \mathrm{diag}(\mu_1,\ \mu_2, \cdots, \mu_s, 0, \cdots, 0)$$

with $\mu_1 \geq \mu_2 \geq \cdots \geq \mu_s > 0$. Then

$$\begin{aligned} & \langle (\bar{M}y + \bar{q}) - (\bar{M}y^* + \bar{q}),\ y - y^* \rangle \\ = & (y - y^*)^\top \bar{M}(y - y^*) \\ = & (y - y^*)^\top P^\top \mathrm{diag}(\mu_1, \mu_2, \cdots, \mu_s, 0 \cdots, 0) P(y - y^*) \\ \geq & (1/\mu_{max})(y - y^*)^\top P^\top \mathrm{diag}(\mu_1^2, \mu_2^2, \cdots, \mu_s^2, 0 \cdots, 0) P(y - y^*) \\ = & (1/\mu_{max}) \|\bar{M}(y - y^*)\|^2. \end{aligned}$$

Since x^* is a solution of (1.1), there exists $\lambda^* = (\lambda_1^*, \lambda_2^*)$ such that $y^* = (x^*, \lambda^*)$ is a solution of (2.2). For any $x \in X$, there exists $\lambda = (\lambda_1, \lambda_2)$ such that $y = (x, \lambda) \in \bar{\Omega}$, and from the definition of (2.2), we have $\langle \bar{M}y^* + \bar{q},\ y - y^* \rangle \geq 0$, and the desired result follows.

For the case that $\mu_{max} = 0$, obviously, $\bar{M}$ is a zero. By (2.2), we also have the desired result follows.

Theorem 3.1 Under Assumption 2.1, for the sequence $\{y^k = (x^k, \lambda_1^k, \lambda_2^k)\}$ generated by Algorithm 3.1, then sequence $\{x^k\}$ terminates in a finite number of steps or converges globally to a solution of the GLCP.

Proof. Since $\gamma > 0$, by Assumption 2.1, (3.1) has an unique solution, so y^{k+1} is uniquely determined. Moreover, (3.1) can be equivalently reformulated as the following inequalities

$$\langle 2(\bar{M} + \frac{1}{2}\gamma I)(y^{k+1} - y^k), y - y^{k+1} \rangle + \langle \bar{M}y^k + \bar{q}, y - y^{k+1} \rangle \geq 0, \ \ \forall y \in \bar{\Omega}. \tag{3.2}$$

Substituting y in (3.2) with y^k, combining (2.1), we have

$$\begin{aligned} f(y^{k+1}) - f(y^k) &= \tfrac{1}{2}(y^{k+1} - y^k)^\top \bar{M}(y^{k+1} - y^k) + \langle \bar{M}y^k + \bar{q}, y^{k+1} - y^k \rangle \\ &\leq \tfrac{1}{2}(y^{k+1} - y^k)^\top \bar{M}(y^{k+1} - y^k) \\ &\quad - \langle 2(\bar{M} + \tfrac{1}{2}\gamma I)(y^{k+1} - y^k), y^{k+1} - y^k \rangle \\ &= (y^{k+1} - y^k)^\top [\tfrac{1}{2}\bar{M} - 2(\bar{M} + \tfrac{1}{2}\gamma I)](y^{k+1} - y^k) \\ &= -(y^{k+1} - y^k)^\top [(3/2)\bar{M} + \gamma I](y^{k+1} - y^k) \leq 0 \end{aligned}$$

Therefore, $f(y^{k+1}) - f(y^k) = 0$ if and only if $y^{k+1} = y^k$, by (3.2), we have that y^k is a solution of (2.2), i.e., the sequence $\{x^k\}$ terminates in a finite number of steps at a solution of GLCP.

In the following convergence analysis, we assume that Algorithm 3.1 generates an infinite sequence, i.e.$f(y^{k+1}) - f(y^k) < 0$.

Consider the function $\Delta(y)$ defined by $\Delta(y) = \Phi(y) + \Psi(y)$, where y^* is a solution of (3.1) and $\Phi(y) = (y - y^*)^\top (\bar{M} + \frac{1}{2}\gamma I)(y - y^*)$, $\ \Psi(y) = \langle \bar{M}y^* + \bar{q}, y - y^* \rangle$. It is easy to deduce $(\gamma/2)\|y - y^*\|^2 \leq \Phi(y) \leq (\mu_{max} + \gamma/2)\|y - y^*\|^2$.

$$\Delta(y) \geq \Phi(y) \geq (\gamma/2)\|y - y^*\|^2 \geq 0. \tag{3.3}$$

For the sequence $\{\Delta(y^k)\}$, set $\Theta(k, k+1) = \Delta(y^k) - \Delta(y^{k+1})$, then a direct computation yields that

$$\begin{aligned}
\Theta(k,k+1) &= (y^k-y^*)^\top(\bar{M}+\tfrac{1}{2}\gamma I)(y^k-y^*)+\langle \bar{M}y^*+\bar{q},y^k-y^*\rangle \\
&\quad -(y^{k+1}-y^*)^\top(\bar{M}+\tfrac{1}{2}\gamma I)(y^{k+1}-y^*)-\langle \bar{M}y^*+\bar{q},y^{k+1}-y^*\rangle \\
&= (y^k)^\top(\bar{M}+\tfrac{1}{2}\gamma I)y^k-(y^*)^\top(\bar{M}+\tfrac{1}{2}\gamma I)y^* \\
&\quad -2\langle(\bar{M}+\tfrac{1}{2}\gamma I)y^*,y^k-y^*\rangle \\
&\quad -(y^{k+1})^\top(\bar{M}+\tfrac{1}{2}\gamma I)y^{k+1}+(y^*)^\top(\bar{M}+\tfrac{1}{2}\gamma I)y^* \\
&\quad +2\langle(\bar{M}+\tfrac{1}{2}\gamma I)y^*,y^{k+1}-y^*\rangle+\langle \bar{M}y^*+\bar{q},y^k-y^{k+1}\rangle \\
&= (y^k)^\top(\bar{M}+\tfrac{1}{2}\gamma I)y^k-(y^{k+1})^\top(\bar{M}+\tfrac{1}{2}\gamma I)y^{k+1} \\
&\quad +2\langle(\bar{M}+\tfrac{1}{2}\gamma I)y^*,y^{k+1}-y^k\rangle+\langle \bar{M}y^*+\bar{q},y^k-y^{k+1}\rangle \\
&= (y^k)^\top(\bar{M}+\tfrac{1}{2}\gamma I)y^k-(y^{k+1})^\top(\bar{M}+\tfrac{1}{2}\gamma I)y^{k+1} \\
&\quad -2\langle(\bar{M}+\tfrac{1}{2}\gamma I)y^{k+1},y^k-y^{k+1}\rangle \\
&\quad +2\langle(\bar{M}+\tfrac{1}{2}\gamma I)(y^{k+1}-y^*),y^k-y^{k+1}\rangle \\
&\quad +\langle \bar{M}y^*+\bar{q},y^k-y^{k+1}\rangle \\
&= (y^k-y^{k+1})^\top(\bar{M}+\tfrac{1}{2}\gamma I)(y^k-y^{k+1}) \\
&\quad +2\langle(\bar{M}+\tfrac{1}{2}\gamma I)(y^{k+1}-y^k),y^*-y^{k+1}\rangle \\
&\quad +\langle \bar{M}y^*+\bar{q},y^k-y^{k+1}\rangle.
\end{aligned}$$

Set $y=y^k$ in Lemma 3.1, then $\langle \bar{M}y^k+\bar{q},y^k-y^*\rangle \geq (1/\mu_{max})\|\bar{M}(y^k-y^*)\|^2$. Hence, if we let $y=y^*$ in (3.2), then

$$\begin{aligned}
&2\langle(\bar{M}+\tfrac{1}{2}\gamma I)(y^{k+1}-y^k),y^*-y^{k+1}\rangle+\langle \bar{M}y^*+\bar{q},y^k-y^{k+1}\rangle \\
&\geq -\langle \bar{M}y^k+\bar{q},y^*-y^{k+1}\rangle+\langle \bar{M}y^*+\bar{q},y^k-y^{k+1}\rangle \\
&= \langle \bar{M}y^k+\bar{q},y^k-y^*\rangle-\langle \bar{M}y^k+\bar{q},y^k-y^{k+1}\rangle+\langle \bar{M}y^*+\bar{q},y^k-y^{k+1}\rangle \\
&\geq (1/\mu_{max})\|\bar{M}(y^k-y^*)\|^2-\langle \bar{M}(y^k-y^*),y^k-y^{k+1}\rangle.
\end{aligned}$$

Thus,

$$\begin{aligned}
\Theta(k,k+1) &\geq \tfrac{1}{2}\gamma\|y^k-y^{k+1}\|^2+(1/\mu_{max})\|\bar{M}(y^k-y^*)\|^2 \\
&\quad -\langle \bar{M}(y^k-y^*),y^k-y^{k+1}\rangle \\
&\geq \tfrac{1}{2}\gamma\|y^k-y^{k+1}\|^2+(1/\mu_{max})\|\bar{M}(y^k-y^*)\|^2 \\
&\quad -(1/\mu_{max})\|\bar{M}(y^k-y^*)\|^2-\tfrac{1}{4}\mu_{max}\|y^k-y^{k+1}\|^2 \\
&\geq \tfrac{1}{2}\gamma\|y^k-y^{k+1}\|^2-\tfrac{1}{4}\mu_{max}\|y^k-y^{k+1}\|^2
\end{aligned}$$

where the last inequality uses $Cauchy-Schwarz$ inequality.

Since $\gamma > \frac{1}{2}\mu_{max}$, we have $\Theta(k,k+1)>0$, and by (3.3), the nonnegative sequence $\{\Delta(y^k)\}$ is strictly decreasing, so it converges, and we get $\Theta(k,k+1)\to 0$ as $k\to\infty$, and thus $\lim_{k\to\infty}\|y^k-y^{k+1}\|=0$. Moreover, $\{\Delta(y^k)\}$ is bounded since it is convergent, and so is $\{y^k\}$ according to (3.3), let $\{y^{k_i}\}$ be a subsequence of $\{y^k\}$ and converges toward $\bar{y}$, by (3.2), we have $\bar{y}$ is a solution of (2.2). The $\bar{y}$ can be used as y^* to define the function $\Delta(y)$: denoted $\bar{\Delta}(y)$, we have

$$(\gamma/2)\|y-\bar{y}\|^2 \leq \bar{\Delta}(y) \leq (\mu_{max}+\gamma/2)\|y-\bar{y}\|^2+\|\bar{M}\bar{y}+\bar{q}\|\|y-\bar{y}\|. \tag{3.4}$$

and we know that $\{\bar{\Delta}(y^k)\}$ also converges, Substituting y in (3.4) with y^{k_i}, we get $\bar{\Delta}(y^{k_i})\to 0(i\to\infty)$, thus, we have $\{\bar{\Delta}(y^k)\}\to 0(k\to\infty)$. By using (3.3) again, we know that the sequence $\{y^k\}$ converges globally toward $\bar{y}$. i.e., the sequence $\{x^k\}$ converges globally to a solution of GLCP.

4 Computational Experiments

In the following, we will implement Algorithm 3.1 in Matlab and run it on a Pentium IV computer. Throughout our computation, **Iter** denotes the number of iterations, f^* is the final value of f in (2.1) when the algorithm terminates, $d(n) = \|x^k - x^{k-1}\|$ where k denotes the number of iterations when the algorithm terminates, and γ denotes the parameter we take.

Our numerical experiment is about the following two sets of problems constructed by Andreani et al in [1] which was also considered by Wang et al in [2]. For simplicity , we make a slight modification.

Example 4.1 Consider the problem of finding $x^* \in R^n$ such that

$$\begin{cases} x \in \mathscr{K} = \{v \in R^n \mid Av \geq 0\}, \\ Nx + d \in \mathscr{K}^\circ = \{v \in R^n \mid v = A^\top \lambda,\ \lambda \in R^s_+\}, \\ x^\top(Nx + d) = 0, \end{cases}$$

where the polyhedral cone $\mathscr{K}$ is generated by s faces whose edges are the following lines:

$$(x, y, z,) = (\rho \cos(\frac{2\pi}{s} i), \rho \sin(\frac{2\pi}{s} i), 1)\tau, \quad \tau \in R,\ i = 1, 2, \cdots, s.$$

Thus, the i-th row of matrix $A \in R^{s\times 3}$ can be computed as

$$\begin{pmatrix} \sin(\frac{2\pi}{s} i)(\cos\frac{2\pi}{s} - 1) - \cos(\frac{2\pi}{s} i)\sin\frac{2\pi}{s} \\ \cos(\frac{2\pi}{s} i)(1 - \cos\frac{2\pi}{s}) - \sin(\frac{2\pi}{s} i)\sin\frac{2\pi}{s} \\ \rho \sin\frac{2\pi}{s} \end{pmatrix}^\top.$$

For each family, we choose $\rho \in \{0.1, 10\}$ and $s \in \{3, 5, 9\}$. The vector d is generated randomly from the interval (-10,10). Matrix N is generated as follows. Denote the orthogonal Householder matrix $Q_{(\cdot)} = I - 2\frac{u_{(\cdot)} u_{(\cdot)}^\top}{\|u_{(\cdot)}\|^2}$, where the components of vector $u_{(\cdot)}$ are generated randomly from $(-1, 1)$. Let D_N be the diagonal matrices whose diagonal elements are generated randomly from $(1, 10)$. We define matrix $N = Q_{NL} D_N Q_{NR}$.

For this problem, we divide the set of test problems into three families:

(1) N is nonsymmetric and indefinite;

(2) N is symmetric and positive definite;

(3) N is symmetric and positive semidefinite.

Obviously, the problems in Families (1) does not satisfy the hypothesis of Theorem 3.1. For each family with different ρ and s, twenty problems are tested with $z = (0, \cdots, 0)^\top$ being the starting point, where μ_{max} is a maximum eigenvalue of $N + N^\top$, $\varepsilon = 10^{-30}$. The numerical results are reported in Table 1.

To take into account the possibility of convergence, we call a case successful if the value of f is less than 10^{-10} within 1000 iterations and we denote by **SP** the successful rate. For all successful cases, **AIter** denotes the average number of iterations, and **ANF** denotes the average number of evaluations for the function f when the algorithm terminates, The numerical results are reported in Table 1, from which we can see Algorithm 3.1 performs well for this set of problems.

Table 1. Average Numerical Results for Example 4.1

s	family	ρ	γ	**AIter**	**SP**	**ANF**
3	(1)	0.1	$(\mu_{max}+0.3)/2$	10	0.55	2.2284×10^{-13}
		10	$(\mu_{max}+2)/2$	57.69	0.65	1.4424×10^{-14}
	(2)	0.1	$(\mu_{max}+2)/2$	220.90	1	1.9710×10^{-13}
		10	$(\mu_{max}+1)/2$	47.32	0.95	6.1555×10^{-13}
	(3)	0.1	$(\mu_{max}+0.2)/2$	182.95	1	1.9710×10^{-13}
		10	$(\mu_{max}+2)/2$	49.74	0.95	6.1565×10^{-13}
5	(1)	0.1	$(\mu_{max}+0.43)/2$	775.64	0.55	5.6263×10^{-14}
		10	$(\mu_{max}+2.5)/2$	90.92	0.65	2.3708×10^{-13}
	(2)	0.1	$(\mu_{max}+0.29)/2$	413.81	0.80	2.7059×10^{-14}
		10	$(\mu_{max}+1)/2$	71.10	1	3.2307×10^{-13}
	(3)	0.1	$(\mu_{max}+0.25)/2$	578.53	0.75	3.0524×10^{-14}
		10	$(\mu_{max}+2)/2$	74.90	1	3.2342×10^{-13}
9	(1)	0.1	$(\mu_{max}+2)/2$	745.33	0.45	4.3307×10^{-15}
		10	$(\mu_{max}+2.85)/2$	194.21	0.70	6.2966×10^{-13}
	(2)	0.1	$(\mu_{max}+2)/2$	786.93	0.70	6.6394×10^{-14}
		10	$(\mu_{max}+2)/2$	354.39	0.90	2.3490×10^{-12}
	(3)	0.1	$(\mu_{max}+2)/2$	752.67	0.60	2.9449×10^{-13}
		10	$(\mu_{max}+2.5)/2$	391.71	0.85	1.1031×10^{-12}

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