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Districting Problem for Several Emergency Service Units and Evaluation of Districts

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Abstract In this paper, we consider districting problems for emergency service systems (such as ambulance systems). In such a service system, typically, N mobile service units cooperate in responding to calls from the public. The assignment of service units is determined by a priority order (such as the order of proximity) for the available service units. We divide the entire service area into several districts by considering the priority order. We give a general formulation for the districting problem and propose a model to obtain the objective for the formulation. The objective function of the formulation is the average response time computed from a stochastic model. The model assumes the continuous-time Markov chain with characteristic of random arrival of calls, assignment of a service unit considering the priority orders, service times depending on a combination of available service units and districts. We give a numerical example of an ambulance system for a city in Japan. Using actual data, we show the usefulness of our formulation and model.

Keywords Public service; Districting Problem; Markov Process.

1 Introduction

This paper focuses on a districting problem for an emergency service system. This service system can exemplify an emergency medical service system (ambulance system), an emergency-repair service system or certain types of home-delivery service systems. These service systems typically own N mobile service units. The service units are stationed at home locations (stations). The home locations are up to N different places in the entire service area. These N service units in cooperation with each other respond to calls from customers.

When a call occurs in the service area, a single service unit is assigned to the call. The assignment of the service unit is determined by using priority orders to dispatch the service unit to the call. For example, the priority order may be determined in accordance with proximity. Using proximity, the first priority service unit would be the nearest service unit, and the second priority service unit would be the second nearest service unit. The first priority service unit would be assigned to the call if it were available. However, if the first priority service unit was busy with a previous call, the first priority service unit would therefore not be able to respond to the new call. In this case, the second priority

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service unit would be assigned to the call. These situations are caused by the fact that service units cannot respond to any new calls when they are on service to a prior call.

We divide the entire service area into several districts considering the priority orders. Berman et al. [2] call the set of these districts the "districting policy." When we determine the districting policy, we need to give a priority order to every call in the entire service area. Calls with the same priority order belong to the same district. In other words, the districting policy is a partition of the entire service area based on the priority orders.

We propose a new model to determine the districting policy for emergency systems. It is very difficult to model these systems because the systems include the following probabilistic features of system operations:

1. Unpredictability of arrival times and of requested sites of calls;
2. Availability of service units;
3. Variability of service times.

The classical location-allocation problems assume deterministic conditions. The typical p-median problem or p-center problem is included in such a problem. However, in the case of these types of service systems, we cannot model with deterministic conditions.

Previous research treated these types of systems as stochastic processes. Carter, Chaiken and Ignall [4] analyzed a case of two service units and two fixed home locations. They assumed a simplified cooperation between the two service units. Larson [6] developed the HYPERCUBE queueing model with N service units. He assumed the same condition as Carter et al[4]. This is a useful model to deal with systems having many service units, because the model can address situations with more than 10 service units. However, these models ignore variations of service time due to variations of travel times. Berman, Larson and Chiu [2] proposed a model that operates as an $M/G/1$ queueing system. Berman and Larson [1] extended the model to deal with more than one service unit. In the model, the entire service area is divided into several service 'territories'. Each service territory has only one service unit. The service territory is an exclusive area for a service unit. Other service units cannot proceed into the exclusive area. In the service territory, a service unit behaves as an independent operation of an $M/G/1$ queueing system. However, there is no cooperation between service units. Inakawa and Suzuki [5] proposed a location problem with several service units. In that paper, districting policy is defined as a Voronoi diagram in which Voronoi generators are home locations of service units.

In this research, we present a mathematical formulation for general districting problems. The formulation enables us to develop general districting policies using integer programming or the other methods. The objective of the formulation is to minimize the average response time. The response time is comprised of the lead-time and the travel time. The lead-time is the time from an occurrence of a call until a service unit leaves its home location to travel to the requested site of the call.

To compute the response time, we propose a model using a Markov process. We assume Poisson arrivals and negative exponential service times. The model has N service units. The model allows no queueing. The service times vary according to the travel times. Travel times depend on both the districts and home locations of the service units. Thus, our model can treat the districting problem based on geographic features. In addition, our model includes the effect of service units being able to cooperate with each other.

It is difficult to solve the formulation because the objective function is very complex. We solve the problem by two steps. In the first step, we solve the formulation with another objective function to develop a districting policy. In the second step, we compute the average response time for given districting policy and compare it with the average response time of the actual districting policy or the other districting policies. Using this procedure we may not obtain the optimal districting policy of all feasible districting policies. However we will at least select better districting policy from among multiple candidates of districting policy using this framework.

In the numerical example, our formulation is applied to the ambulance system of Seto City in Japan. Seto City has three home locations with four ambulances. The city has 361 demand points. In the first step of this numerical example, we develop a new districting policy based on the proximity orders. The districting policy is computed using the framework of our formulation. In the second step, we compute the average response time that is the objective function and compare the new districting policy with the actual districting policy. Evaluating the districting policies by average response time, we can select the better districting policy from among these two districting policies.

In the next section, we give a general formulation to determine a districting policy. In the third section, we propose a stochastic model to obtain the average response time. In the fourth section, we provide a numerical example of our formulation and considerations for this problem.

2 Formulation

To provide a detailed explanation about districts, we present an overview of general emergency service systems. The chain of events for an incident includes the following operations:

1. Incident detection and reporting;
2. Service unit dispatching;
3. Actual treatment by the service unit's crew.

In addition, a service unit having finished the third operation will return to its home location, and make ready for next incident. Emergency service systems principally repeat those operations in this order.

The role of the districts is of concern in the second operation. Suppose that a call occurs in a district, then, in the second operation, we need to determine the available service unit with the highest priority order so that it can be dispatched. For that purpose, we check the available service unit by the priority order of the district which is decided in advance for each district.

We present a simple example for districts and their priority orders. On a Euclidean plane, if we divide the entire service area into the districts based on the proximity, districts become a higher order Voronoi diagram in which Voronoi generators are home locations of service units. Figure 1 is an example with three home locations $S_i, i = 1,2,3$. The figure is called the Voronoi diagram. It shows the nearest area for each home location. Furthermore, Figure 2 calls the third (or second) order Voronoi diagram, and this is a simple example of a districting policy with three home locations. In Figure 2, the sequential numbers in parentheses are the priority order numbers used to dispatch service units to

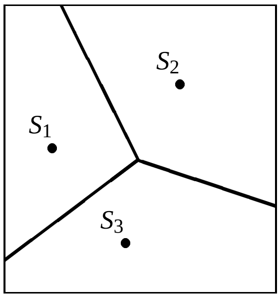

Figure 1: Voronoi diagram

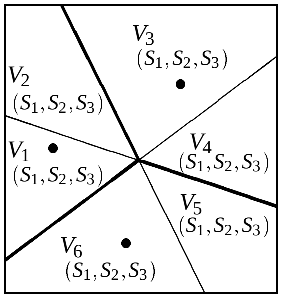

Figure 2: Districts

calls. We find 6 districts with unique priority orders. This number of districts equals the total number of permutations of home locations. In fact, the number of types of districts is ${}_3P_3 = 3! = 6$. If more than one service unit stations at a home location, the number of districts does not change.

That is not the only way to divide the entire service area into districts. We can easily imagine a number of districting policies. For example, when there are many home locations, it might not be necessary to consider the full permutation of all home locations. Let us consider an example of a specific emergency system. In this system, it has been empirically found that customers are always served by only the nearest two service units. This system may have a very low incidence of calls. In this case, it might be sufficient to regard this as a two-permutation (a permutation of the n objects taken 2 at a time) problem. Let us consider another example in which there is large bias in distribution of customer demand. Then, a districting policy based on proximity may not be appropriate. In this case, a strategic districting policy considering the bias must be more appropriate.

In this manner, many acceptable districting policies are feasible. However, if the types of districts are defined, a decision of districting policy might be regarded as a problem of assigning a district number to each demand point. This problem can be formulated in general as an integer programming. Let

$$x_{kj} = \begin{cases} 1, & \text{if demand point } k \text{ is assigned to district } V_j, \\ 0, & \text{otherwise,} \end{cases} \quad \text{for all } k,\ j.$$

Then a districting policy can be denoted by $\mathbf{X} = [x_{kj}]$. If there are J districts and K discrete demand points, the formulation is as follows:

$$\text{Minimize} \qquad f(\mathbf{X}) \tag{1}$$

$$\text{Subject to} \quad \sum_{j=1}^{J} x_{kj} = 1, \qquad k = 1,2,\ldots,K,$$

$$x_{kj} \in \{0,1\}, \quad k = 1,2,\ldots,K,\ j = 1,2,\ldots,J.$$

In the next section, we describe the concrete objective function and a method to compute it.

3 Model and Objective

In this section, we propose a model to compute the average response time that is the objective function. The essential point of this modeling is to incorporate the geographical information and statistical characteristics.

One of the basic pieces of geographical information is the distribution of customers. We assume that customers are geographically distributed on discrete demand point k with ratio h_k ($\sum_{k=1}^{K} h_k = 1$). Calls can occur only at the demand points within the entire service area, each demand point k generates calls according to a Poisson process with rate λh_k. We assume that calls independently occur at these demand points.

We assume that the service time of a service unit for a call is the total time that the service unit is occupied by the call. The service time is comprised of the response time and the on-site service time. The response time includes the lead-time l_i and the travel time t_{ik}. The lead-time l_i is the time from the occurrence of a call until server i leaves its home location to travel to the requested demand point. The travel time t_{ik} is the time necessary to move from the home location of service unit i to demand point k. For example, the travel time t_{ik} might be equal to $d(i,k)/v_{ik}$ by means of travel distance $d(i,k)$ and a travel speed v_{ik}. Following the response time, there is an on-site service time. During the on-site service time, paramedics may give a patient first aid. In addition, the on-site service time includes the time required for carrying the patient to a hospital and the time for the unit to return to its home location from the hospital.

Since customers who call for service are requesting to receive first aid as soon as possible, we set the average response time as our objective function. Then, our purpose was to determine the districting policy that minimizes the average response time. In the following section, we present a model to obtain the average response time.

3.1 Model

For the modeling of emergency systems, the time for the occurrence of calls and for the completion of service is important. We discuss the arrival and service processes of calls.

Suppose that a call occurred from a demand point and the nearest service unit responded to the call. After the call, if another call occurs from the same demand point, the same service unit or another service unit would respond to the new call. When the same service unit has not finished its service for the previous call, another service unit would respond to the new call. Thus, calls do not always receive service from the same service unit even if those calls originate from the same demand point. This is a difficult point in modeling emergency systems.

On the other hand, if the situation of available service units is given, the service unit is uniquely determined by the priority order of the demand point. For example, we consider two demand points that possess the same priority order. Suppose that a call occurs from one demand point among these two demand points. Then, we can uniquely determine the service unit to be dispatched regardless of the kind of demand points, because these two demand points possess the same priority order. Even if the situation of available service units is changed, this property is kept.

Using this property of assignment, we merge the arrival processes. Poisson processes of each demand point are merged into a Poisson process if they belong to the same district.

This merging means the superposition of Poisson processes. We perform the superposition on each district. On the merged Poisson process, calls are generated from the gravity center of demands in a Poisson manner with rate λ_j. In particular,

$$\lambda_j(\mathbf{X}) = \lambda h^j(\mathbf{X}), \quad \text{where} \quad h^j(\mathbf{X}) = \sum_{k=1}^{K} h_k x_{kj}.$$

We merge service time distributions as well as arrival processes as described above. Service time is comprised of the response time and the on-site service time. The response time includes travel time. When a district is given, the average travel time to any calls within the district is the travel time from a home location to the gravity center of demand points in the district, because sites of calls within the district depend on the intensities of Poisson processes on demand points. When service unit i respond to any call within the district j, the average travel time τ_{ij} is

$$\tau_{ij}(\mathbf{X}) = \sum_{k=1}^{K} t_{ik} \frac{h_k}{h^j(\mathbf{X})}.$$

The lead-time l_i included in the response time would not change by the merging. As a consequence, the response time when server i responds to a customer from district j is

$$R_{ij}(\mathbf{X}) = l_i + \tau_{ij}(\mathbf{X}).$$

The merging would also make no difference for the on-site service time, because the on-site service time would depend largely on the patient's disease or injury symptoms. In fact, in some cases, based on the patient's symptom, it may be not necessary to give any treatment, or it may be necessary to carry the patient to a far-distant hospital that owns updated equipment. We denote the average of on-site service times as A.

When service unit j responds to a call within the district i, the service time is $R_{ij}(\mathbf{X}) + A$. We assume that the service times when server i responds to a call from district j are exponentially distributed with mean μ_{ij}^{-1}, where

$$\mu_{ij}(\mathbf{X}) = [R_{ij}(\mathbf{X}) + A]^{-1}.$$

Since we assume Poisson arrivals and negative exponential service times, this model is thus a finite state continuous-time Markov process.

3.2 Objective

The formulation given in Section 2 is a general form for the districting problem. In this section, we describe a concrete computational method of the objective function that is the average response time. The objective function (1) is expressed by the following.

$$f(\mathbf{X}) = W(\mathbf{X})\mathbf{P}. \tag{2}$$

$W(\mathbf{X})$ in equation (2) is a vector whose elements are the average times on each state of a continuous-time Markov chain, adopting the districting policy $\mathbf{X}$. $\mathbf{P}$ in equation (2) is a stationary distribution satisfying next equations.

$$Q(\mathbf{X})\mathbf{P} = \mathbf{0}, \quad (3)$$
$$\mathbf{IP} = 1, \quad (4)$$
$$\mathbf{I} = (1,1,\ldots,1).$$

$Q(\mathbf{X})$ in equation (3) is a transition matrix adopting the districting policy $\mathbf{X}$. $\mathbf{I}$ in equation (4) is a row vector whose all elements are 1. The equation (4) is a condition that $\mathbf{P}$ is a probability distribution.

3.3 Matrix Generating Methods

When $\lambda_j(\mathbf{X})$ and $\mu_{ij}(\mathbf{X})$ are given, the following method enables us to generate the transition matrix $Q(\mathbf{X})$. Suppose that J is the number of districts, and N is the number of service units.

Step 1. $s = 0$. $\hat{Q} = 0$.

Step 2. E = identity matrix of $(J+1)^s$.

Step 3. $L_j = \lambda_j(\mathbf{X})E$. $M_{N-s,j} = \mu_{N-s,j}(\mathbf{X})E$.

Step 4. Delete elements of L_j if there is no transition.

Step 5. Generate the matrix as follows, and update $\hat{Q}$.

$$\hat{Q} = \begin{pmatrix} \hat{Q} & L_1 & L_2 & \cdots & L_J \\ M_{N-s,1} & \hat{Q} & & & \\ M_{N-s,2} & & \hat{Q} & & \mathbf{0} \\ \vdots & & & \ddots & \\ M_{N-s,J} & & \mathbf{0} & & \hat{Q} \end{pmatrix}. \quad (5)$$

Step 6. $s = s+1$. If $s < N$, go to step 2.

Step 7. Denote ${}^t\hat{Q}$ by $Q(\mathbf{X}) = [q_{ij}]$, and $q_{ii} = -\sum_j q_{ij}$.

The subscript t in ${}^t\hat{Q}$ represents transpose.

To generate $W(\mathbf{X})$, we define a priority order list PM $= [b_{j,u}]$, where $b_{j,u}$ is the number of the uth priority service unit for call from district j. For example, PM for Table 1 in section 4 is

$$\mathrm{PM} = {}^t\begin{bmatrix} 1 & 1 & 2 & 2 & 3 & 3 \\ 3 & 2 & 1 & 3 & 2 & 1 \\ 2 & 3 & 3 & 1 & 1 & 2 \end{bmatrix}.$$

We denote a base J number having N digits by t, and s_i is ith digit of t. Initially, $W(\mathbf{X})$ is set a zero vector having the order of $(J+1)^N$. The tth element of $W(\mathbf{X})$ is denoted by W_t. Then, the following method computes each value of W_t.

Step 1. $t = 00\cdots0$.

Step 2. $j = 1$.

Step 3. $h = 1$.

Step 4. $i = b_{j,h}$.

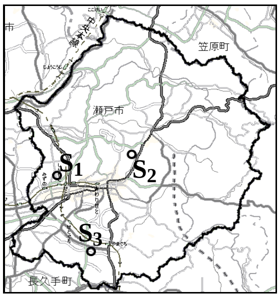

Figure 3: Traffic network of Seto City

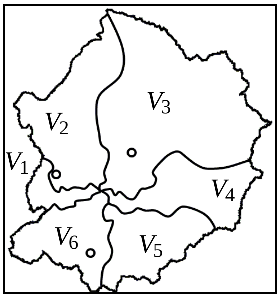

Figure 4: Actual districting policy of Seto City

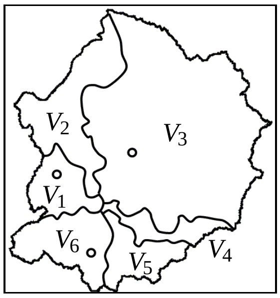

Figure 5: New districting policy $\mathbf{X}^G$

Step 5. If $s_{N+1-i} \neq 0$, $h = h+1$ and go to step 4. Otherwise $W_t = W_t + h^j \tau_{ij}$.

Step 6. If $j < J$, $j = j+1$ and go to step 3.

Step 7. $t = t+1$. If $t \neq (J+1)^N - 1$, go to step 2.

Step 8. $W_t =$ loss cost.

In the iteration between step 4 and 5, a district is fixed, and an available service unit is sought in the priority order. In step 7, if j is less than J, go to step 3 and consider a next district. In step 8, if t is less than $(J+1)^N - 1$, go to step 2 and consider a next state of the Markov chain. The loss cost in Step 9 is a given constant number. It might be represented by the average travel time from an adjacent community's home location to the center of gravity for the entire service area.

4 Numerical Example

In this section, we introduce an application of our formulation for the ambulance system of Seto City in Japan. Seto City is a medium scale city located in Chubu area of Japan. The population of the city is about $130,000$, and the width and the breadth of the city are about 13 km and 12 km respectively. Figure 3 is an overview of Seto City. In the city, there were 4,733 calls in 2005, and the number of calls increases every year. This city has three home locations (ambulance stations) with four ambulances (one home location has two ambulances). Figure 3 shows these situations. Home location S_1 has two ambulances. We set each of the Zip-code areas in the city as a unit of demand points, because the unit of the actual districting policy is done that way. Thus, there are 361 demand points.

Seto City already has the own districting policy shown in Figure 4. As in the case of our model in Section 3, assignments of ambulances are determined by the priority orders of these districts. The number of districts is six, which is the total number of full permutation of home locations. The priority orders of those districts are tabulated in Table 1. In this numerical example, we develop a new districting policy using a simple process. We determine a more appropriate districting policy from the actual districting policy and the new districting policy.

The necessary information for computation is listed as follows:

Table 1: Priority orders

District	1st 2nd 3rd
V_1	$S_1 \to S_3 \to S_2$
V_2	$S_1 \to S_2 \to S_3$
V_3	$S_2 \to S_1 \to S_3$
V_4	$S_2 \to S_3 \to S_1$
V_5	$S_3 \to S_2 \to S_1$
V_6	$S_3 \to S_1 \to S_2$

Table 2: Travel speed

	V_1,V_2	V_3,V_4	V_5,V_6
S_1	33.76	39.03	33.45
S_2	43.13	40.70	43.53
S_3	38.92	42.38	37.56

(km/hour)

Table 3: Average Response Times

Year	Observed	Model
2001	5.1657	5.2661
2002	5.3008	5.2980
2003	5.8669	5.8190

(Minute)

- The entire arrival rate λ (average inter-arrival time λ^{-1});
- Lead-time l_i;
- On-site service time A;
- Travel time t_{ik}:
  - Distance $d(i,k)$ from home location i to demand point k; Travel speed v_{ij};
- The loss cost;
- The distribution of population h_k.

Some of those parameters require actual emergency medical service data, and we obtain this data from the emergency medical service data of Seto City in 2001-2003. The data contains more than 11,000 incident (call) records. Lead-times l_i, travel times t_{ik} and an on-site service time A are obtained from the data. In addition, we obtain the average inter-arrival time λ^{-1} from the data.

The loss cost is the time required when all ambulances are busy. That is the time from when a call occurs until an adjacent community's ambulance reaches the requested site in Seto City. The actual data for loss cost would be found in the data from the adjacent communities that responded to the call from Seto City. Since we do not have access to that data and only have data from Seto City, we cannot compute the actual loss cost. Therefore, instead of computing the actual loss cost, we make the assumption that the loss cost is equivalent to the average time from when a call occurs in adjacent communities until Seto City's ambulance reaches the site of the call.

Most of the travel times t_{ik} can be obtained from the data. However, the data about combinations of all demand points and all home locations is incomplete. In addition, at some demand points, there is only data on one or two calls (incidents). In those cases, we compute t_{ik} using results of the regression analysis if the data did not contain a sufficient number of records. We assume that the travel speed of ambulance i is equal to v_{ij} when ambulance i respond to a demand point in district j. We measure the travel distances using ZENRIN 96. The results of regression when ambulance i travels to districts j is in Table 2. We suppose that the district 1 and 2 has the same travel speed. This assumption

Table 4: Result of computation

	average response time
Actual districting policy	5.7825
New districting policy ($\mathbf{X}^G$)	5.7171

Table 5: Probability to response

	1st	2nd	3rd	loss
Actual districting policy	0.8858	0.1027	0.0083	0.0032
New districting policy ($\mathbf{X}^G$)	0.8929	0.0950	0.0088	0.0033

is also made in the case of district 3 and 4, and district 5 and 6. This assumption assures a sufficient number of records and decreases our computational effort. Finally, data of population distribution is downloaded from website of Seto City [9].

We attempted the experimental reproduction of the observed average response time using our model. We computed the average response time for each year by means of changing the entire arrival rate λ and the distribution of population h_k. Table 3 shows the comparisons of the observed average response time and the average response time that were computed from our model. We find that the errors are within allowable limits.

In March 2003, a new tunnel was built in the city. Thus, we considered a situation that the new tunnel can be used. Updating the travel-times including the information of the new tunnel, we conducted a numerical example for the districting problem.

4.1 New Districting policy

We develop a new districting policy using a simple method. In this simple method, we divide the entire service area based on proximity. Berman et al. [1] call this method a proximity design. In other words, the proximity design is Greedy approach. Proximity design can be achieved by various methods. With the aid of the formulation in section 2, we formulate the proximity design as follow.

$$
\begin{aligned}
\text{Minimize} \quad & \sum_{k=1}^{K}\sum_{j=1}^{J} x_{jk} \sum_{i=1}^{M} a_{ij} t_{ik} \\
\text{Subject to} \quad & \sum_{j=1}^{J} x_{kj} = 1, \qquad \forall\, k = 1,\ldots,K, \\
& x_{kj} \in \{0,1\}, \quad \forall\, k = 1,\ldots,K,\ j = 1,\ldots,J,
\end{aligned} \tag{6}
$$

where a_{ij} is a weight of travel time for each home location.

After the computation, the consequence of $\mathbf{X} = [x_{ik}]$ is the districting policy based on proximity. We denote this new districting policy as $\mathbf{X}^G$. Figure 5 shows an overview of the new districting policy $\mathbf{X}^G$.

We compare the new districting policy $\mathbf{X}^G$ with the actual districting policy of Seto City. Using the model in Section 3, we compute the average response time on each districting policy. The results of computation are tabulated in Table 4. When the new districting policy $\mathbf{X}^G$ is adopted, the average response time is 5.7171 minutes. When using the actual districting policy, it is 5.7825 minutes. By adopting the new districting policy, we obtain a shorter average response time. The new districting policy decreases the average response time by 0.0653 minutes (3.92 seconds). Therefore, the optimal districting policy from among two districting policies is determined to be the new districting policy $\mathbf{X}^G$.

4.2 Consideration

In this subsection, we discuss the difference of these average response times. In general, a difference of 3.92 seconds may not seem to be a big deal. However, since the number of calls (incidents) in Seto City is $4,733$, the cumulative difference may exceed five hours.

Specifically, the reduction of the response time is caused by two factors. One factor is the direct modification of the district for each demand point. The new districting policy changes the actual district of some demand points to a district that is more appropriate. The actual districting policy was made based on the experience of paramedics, in fact, neither exact distances nor travel times were measured. In addition, the actual districting policy was made several decades ago. Thus, customers whose location changed districts would obtain shorter response times.

The other factor is the assignment to higher priority ambulances. This benefit is the result of a modification of system efficiency. Therefore, every customer in the city may receive this benefit. In fact, the new districting policy increases the probability that a customer will be assigned to higher order priority ambulances. For example, the probability that a customer is assigned to the first priority ambulance is increased by 0.0071. Difference of the probability implies that people who are assigned to the first priority ambulance is increased by 34 per year. We have concrete examples for 34 people. For a demand point, the response time from each home location is 4.75, 9.47 and 10.14 minutes respectively. By changing the assigned ambulance from the second priority ambulance to the first priority, one reduces the response time by 4.72 minutes. This difference could possibly save patient lives.

5 Summary and Conclusions

In this paper, we provided a formulation of the districting problem for emergency service systems. We can determine a more appropriate districting policy by the average response time. The average response time derived from Markov model is computed under realistic assumptions, such as the arrival process of calls and the availability of service units. Since the average response time is an easily understood index, computational results can be used to obtain consent from actual decision makers. In fact, we proposed the result of the numerical example in Section 4 to the Shoubou-Honbu of Seto City. Although the suggestion was not adopted in its entirety, it was used as a reference material for the restricting of the ambulance system.

In this modeling, we assumed a medium-scale service system. Many previous models assume large-scale service systems. In order to execute large-size models, these models sacrifice detailed real behavior. However, in the case of medium-scale systems, we can propose models that are more detailed. There are many more medium-scale service systems than large-scale ones. We believe that our model can help these medium-scale service systems.

In the numerical example in Section 4, we developed a districting polity based on proximity. However, other districting policies could also be proposed. For example, home location S_1 in Seto City has two ambulances. A difference in the number of service units would have an affect on determining districting policy. To be able to show the optimality of a districting policy from among any feasible districting policies requires further discussion.

In addition, we are interested in districting policy when home locations are changed. In order to try to solve this problem, we would need an appropriate traffic network with time speeds. The traffic network is important because setting a new home location requires computing of new travel times rather than using observed data. Construction of an appropriate traffic network is currently underway and will be reported on later papers. On the other hand, we have confirmed the existence of a more effective districting polity than proximity design, though without showing optimality. However, an efficient algorithm to create an optimal districting policy is desirable.

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