The 7th International Symposium on Operations Research and Its Applications (ISORA'08)
Lijiang, China, October 31–Novemver 3, 2008

A Cointegration Analysis of the Relationships among Tax Revenue and Macroeconomic Factors in China*

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Abstract Using macroeconomic factors to forecast tax revenue is an important means of tax revenue forecasting, and it is also a challenging issue in China to which State Administration of Taxation pays close attention. As a prerequisite, the relationships among tax revenue and certain macroeconomic factors should be examined. In this paper, interactions among tax revenue, GDP, price level, and external trade in China are investigated. With the benefit of cointegration analysis, the long-run equilibrium conditions among tax revenue and these macroeconomic factors are proposed. Based on these cointegrating relations, a vector error correction (VEC) model is then estimated. The results show both long-run and short-run relationships among total tax revenue, GDP, price level, and external trade in China, which could help to promote further studies concerning tax revenue analysis and forecasting using macroeconomic factors.

Keywords cointegration, vector error correction (VEC), tax revenue, macroeconomic factors

1 Introduction

During the past 20 years, tax revenue has become the largest source of state fiscal revenue in China, accounting for 88.9 percent of state fiscal revenue in 2007, or 4,561,299 billion yuan. Tax revenue forecasting then plays a significant role in government budget. As a prerequisite for accurate and reasonable forecasting, factors affecting tax revenue and relationships among their performances should be examined carefully. Since tax revenue is a macroeconomic factor per se, it is closely related with some other specific macroeconomic factors. In China, these factors include GDP, price level, external trade, and so on, indicated by Professor Zhang, former Director of Scientific Institute of State Administration of Taxation [1, 2].

Through participation in the project of State Administration of Taxation in 2007, we realize again the necessity and importance of analysis of link among tax revenue and these factors. Further, we aware that more effective and novel analytical methods that

*This paper is supported by State 973 Program (2006CB701306), and Key Laboratory of Management, Decision and Information Systems, CAS.
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have been increasingly used in similar fields abroad, should be introduced and applied in the combination with Chinese economic situation. For example, cointegration and error correction theory which is suited to capture both long- and short-run relationships among economic factors has been pervasively used in various cases in western countries since it was proposed by Eagle and Granger. However, only a few recent studies in China have started to use this method to tax issues [3, 4, 5, 6].

Therefore, the primary focus of our study is to investigate the interactions among tax revenue, GDP, price level, and external trade. A secondary purpose is to apply cointegration and vector error correction (VEC) modeling techniques to Chinese tax revenue analytic case. The outline for this paper is as follows. Section 2 deals with data issues and unit root tests. Section 3 uses cointegration to analysis the long-run equilibrium among tax revenue and these macroeconomic factors. A VEC model is estimated in Section 4. The paper is concluded in Section 5.

2 Data and unit root tests

Annual data are obtained from *China Statistical Yearbook* [7, 8], over the period of 1985–2006. Data used in our study start from 1985 because total tax revenue figures from 1985 are not comparable with those in previous years, due to institutional tax reform in China during 1980s [8].

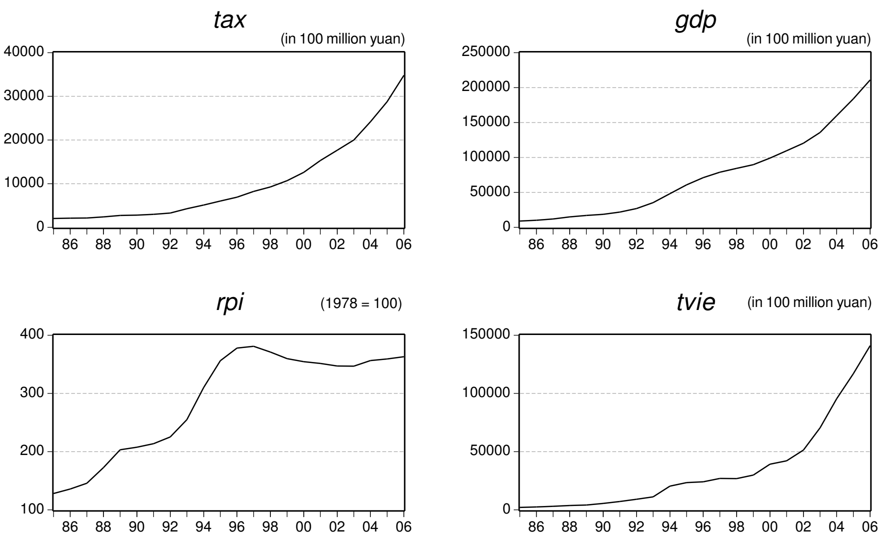

Figure 1: *tax*, *gdp*, *rpi*, and *tvie* in value terms in China (1985–2006)

Four variables (or called series) are considered: total tax revenue (*tax*), GDP (*gdp*), retail price index (*rpi*), and total value of imports and exports (*tvie*). The last two ones reflect price level and external trade respectively. Since price level is included in our study, other three variables are calculated at current prices. The trends of these variables from 1985 to 2006, plotted in Figure 1, suggest the possible existence of strong links among them. In particular, *tax* tracks closely *gdp* and *tvie*. Moreover, while *rpi* sharply rose from

1993 to 1996, reached a peak in 1997, and then started with a slow decline, gdp and $tvie$ fluctuated to some extent as well. But it is unclear from Figure 1 the reason that tax did not experience similar changes throughout this period. These observations motivated us to explore the relationship among these variables with the benefit of cointegration theory.

Unit root tests which examine the integration properties of the variables are necessary before estimating the cointegrating equation(s). A variable x is integrated of order d, denoted by $x \sim I(d)$, if x has d unit roots, or equivalently, the dth differences of x becomes stationary [9]. In this paper, Augmented Dicker-Fuller (ADF) tests are used to investigate if all these variables are integrated of the same order. Results of the tests presented in Table 1 show that these four variables are all $I(2)$ series.

Table 1: ADF unit root tests

H_0: Variable x has a unit root.

Variables	ADF(c, t, p)	Test critical values	
		1% level	5% level
tax	ADF(c, t, 0) = 8.458667	-4.467895	-3.644963
gdp	ADF(c, t, 2) = 0.632410	-4.532598	-3.673616
rpi	ADF(c, t, 2) = -1.001784	-4.532598	-3.673616
$tvie$	ADF(c, t, 0) = 3.762624	-4.467895	-3.644963
Δtax	ADF(c, t, 0) = 0.172537	-4.498307	-3.658446
Δgdp	ADF(c, t, 1) = -1.721439	-4.532598	-3.673616
Δrpi	ADF(c, 0, 1) = -2.773259	-3.831511	-3.029970
$\Delta tvie$	ADF(c, t, 0) = -1.679174	-4.498307	-3.658446
$\Delta^2 tax$	ADF(c, 0, 0) = -3.536862*	-3.831511	-3.029970
$\Delta^2 gdp$	ADF(0, 0, 0) = -2.487641*	-2.692358	-1.960171
$\Delta^2 rpi$	ADF(0, 0, 1) = -3.746939**	-2.699769	-1.961409
$\Delta^2 tvie$	ADF(0, 0, 0) = -4.410140**	-2.692358	-1.960171

Note: Whether intercept (c) or trend (t) are included in ADF tests are decided based on Gao's suggestion [10]. The number of lags (p) in ADF tests are chosen based on Schwarz information criterion (SIC), MAXLAG = 4.

* Significant at the 5% level.

** Idem., 1%.

3 Long-run equilibrium: cointegration

Cointegraton tests using the methodology developed by Johansen [11, 12] are performed in our study. The simple form of Johansen's method is stated as follows.

Consider a VAR (vector autoregression) of order p :

$$y_t = \sum_{i=1}^{p} A_i y_{t-i} + Bx_t + \varepsilon_t \tag{1}$$

where y_t is a k-vector of non-stationary $I(1)$ variables, x_t is a d-vector of deterministic variables such as trend and constant, and ε_t is a k-vector of innovations. It could be rewritten as:

$$\Delta y_t = \Pi y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + Bx_t + \varepsilon_t \tag{2}$$

where $\Pi = \sum_{i=1}^{p} A_i - I$, $\Gamma_i = -\sum_{j=i+1}^{p} A_j$. The number of cointegrating relations among the components of the vector y_t is represented by the rank of Π. If Π has rank $r < k$, then there exist $k \times r$ matrices α and β each with rank r such that $\Pi = \alpha\beta'$ and $\beta y_t \sim I(0)$ [9, 11]. So, r is also called the conintegrating ranks, β is the cointegrating vector, and the components of α are the adjustment parameters. Johansen's method is to estimate the matrix Π based on an unrestricted VAR and to test the number of non-zero eigenvalues of Π (which equals r) applying trace or maximum eigenvalue statistics.

Given the small number of observations in our study, a cointegration test with one lag in the level series is used. Notice the appearance of each variable, a quadratic trend case is chosen among five trend cases considered by Johansen [12]. Table 2 reports the test results. Both trace and maximum eigenvalue tests indicate two cointegrating equations at the 5% level. The estimation of β and α are summarized in Table 3, which are also applied to VEC modeling in Section 4.

Table 2: Johansen cointegration test

Hypothesized No. of CE(s)	Eigenvalue	Trace statistic	Max-Eigen statistic
None*	0.946153	99.54366	61.35383
At most 1*	0.718004	38.18983	26.58312
At most 2	0.344232	11.60671	8.860904
At most 3	0.122565	2.745803	2.745803

* Significant at the 5% level.

Table 3: Restricted estimation of cointegrating and adjustment coefficients.

	Cointegrating Eq.1	Cointerating Eq.2
Normalized cointegrating coefficients β		
tax	1.000000	0.000000
gdp	0.000000	1.000000
rpi	0.835221	-298.8171
$tvie$	-0.180966	-0.241814
Adjustment coefficients α		
Δtax	-0.388744	0.049688
Δgdp	-2.839178	0.228138
Δrpi	-0.008967	0.000281
$\Delta tvie$	-0.267771	0.218230

Using standard notation, the long-run equilibrium conditions are finally stated ($t = 1985, \cdots, 2006$):

$$z_{1t} = (tax)_t + 0.835221 * (rpi)_t - 0.180966 * (tvie)_t - 370.9212 * (trend)_t + 131.6052 \tag{3}$$

and

$$z_{2t} = (gdp)_t - 298.8171 * (rpi)_t - 0.241814 * (tvie)_t - 3233.2355 * (trend)_t + 60389.00 \tag{4}$$

where $(trend)_t = t - 1985$ in the year t.

The signs of all parameters of the long-run cointegrating relations are as expected and their size are reasonable. In particular, according to Eq.(3), a 10% permanent increase in *rpi* causes a 8.35% reduction in *tax*. Similarly, a permanent 10% increase of *tvie* is associated with a 1.81% growth in *tax*. Eq.(4) indicates the relationship among *gdp*, *rpi*, and *tvie*. An increase in the level of either price or external trade causes GDP to rise. From Eq.(3) and Eq.(4), *rpi* seems to affect *tax* and *gdp* not only in different directions but also to very different degrees. This finding, along with essential economic theories, may be attributed to explain the "insensitivity" of *tax* (compared with *gdp* or *tvie*) to price fluctuation mentioned in Section 2.

4 Short-run dynamics: a vector error correction (VEC) model

Based on the results of the cointegration test above, a VEC model with no lagged first difference terms is estimated. In this paper, how tax revenue is influenced by other macroeconomic factors is paid more attention. Therefore, only the equation for Δtax is presented below.

$$\Delta(tax)_t = -0.388744 * z_{1t-1} + 0.049688 * z_{2t-1} - 1026.101 + 235.1388 * (trend)_t \quad (5)$$

z_{1t} and z_{2t} are calculated as Eq.(3) and Eq.(4) respectively. Detailed information is shown in Figure 2 and Table 4. Although the equation has relatively simple structure, its fitting performance and other statistical properties are satisfying.

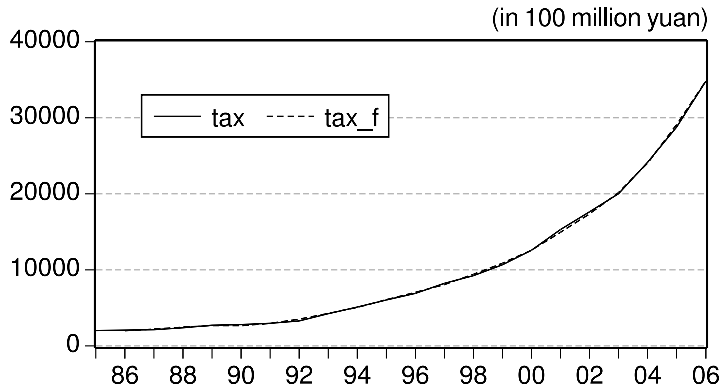

Figure 2: the real and fitted value of tax revenue in China (1985–2006)

5 Conclusion

This paper focuses on the relationships among total tax revenue, GDP, price level, and external trade in China. Considering the time-series properties of the variables, we use cointegration analysis to indicate long-run equilibrium conditions among tax revenue and other three macroeconomic factors in China. According to these cointegrating relations, a VEC model is then estimated, from which we provide an equation for fitting the value of total tax revenue. The results suggest strong links among tax revenue and certain macroeconomic factors in China, which could be applied to further studies concerning

Table 4: Equation for $\Delta(tax)_t$.

	Coefficient	Standard error	t-statistic	Prob.
z_{1t-1}	-0.388744	0.05930	-6.55544	0.0000
z_{2t-1}	0.049688	0.00342	14.5107	0.0000
constant	-1026.101	98.2845	-10.4401	0.0000
$(trend)_t$	235.1388	7.82735	30.0407	0.0000
R-squared	0.985220	Log likelihood	-140.5762	
Adj. R-squared	0.982612	Akaike AIC	13.76916	
F-statistic	377.7366	Schwarz SIC	13.96812	
Prob(F-statistic)	0.000000			

tax revenue forecasting. Moreover, cointegration and error correction theory are shown in this paper appropriate and effective for tax revenue analysis and forecasting using Chinese economic situation.

References

[1] Zhang, P.S.: Analysis of tax revenue and economic situation in 2002–2003 (excerpt), http://www.china.com.cn/zhuanti2005/txt/2002-12/20/content_5249940.htm, 2002.

[2] Li, B.: Growth of Tax Revenue and Trend of Economic Development— Interviewing Mr. Zhang Peisen, a research fellow at Scientific Institute of State Administration of Taxation, *China Economist*, No. 3, pp. 6–7, 2006.

[3] Wang, N.J., Li, G.F.: A Research into the Application of the Theory of Co-integration in the Calculation of the Tax Revenue, *Shangdong Economy*, Gen. 113, No. 6, pp. 21–23, 2002.

[4] Xue, W., Zhang, M.: Granger cause and cointegration test on China's tax revenue, *Journal of Central University of Finance and Economics*, No. 11, pp. 16–19, 2005.

[5] Zhang, S.Q.: Research on tax prediction error correction model based on cointegration theory, *Journal of South China Normal University (Natural Science Edition)*, No. 1, pp. 9–14, 2006.

[6] Guo, J.: A Conintegration Analysis on Tax, Government Spending and Growth of Economy in China, *Research on Financial and Economic Issues*, Gen. 276, No. 11, pp. 82–86, 2006.

[7] National Bureau of Statistics of China, *China Statistical Yearbook*, 1994.

[8] National Bureau of Statistics of China, *China Statistical Yearbook*, 2007.

[9] Engle, R.F., Granger, C.W.J.: Co-integration and error correction: representation, estimation and testing, *Econometrica*, Vol. 55, No. 2, pp. 251–276, 1987.

[10] Gao, X.M.: *Econometric analysis and modeling—Eviews Applications and Examples*, Tsinghua University Press, 2006.

[11] Johansen, S.: Estimation and hypothesis testing of cointegration vectors in Gaussian vector autoregressive models, *Econometrica*, Vol. 59, No. 6, pp. 1551–1580, 1991.

[12] Johansen, S.: *Likelihood-based inference in cointegrated vector autoregressive models*, Oxford University Press, 1995.