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# Traffic jamming in disordered flow distribution networks

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**Abstract** Traffic transportation can often been observed in many realistic networks, such as airport networks, highway road networks, resistor networks. In this study, we investigate the effect of disorder (the flow distributions) on the jamming degree in different networks by the method of the gradient flow. With the new definition of the jamming coefficient in weighted networks, the numerical results show that the more disordered the flows distribute, the higher level of congestion the network will be. The results may provide some insights on the control strategy of traffic flow in networks.

**Keywords** Complex system, network dynamics

## 1 Introduction

During the last few years, complex network research has attracted much interests from socialist, biologists, mathematicians, and physicists communities [1, 2, 3, 4, 5, 6]. One of the most interesting topics in complex network research is the transportation ability involved in networks [7].

Since Toroczkai [8, 9] proposed the gradient networks to describe the flow in networks, much work has been done to explore the jamming degree in scale-free (SF) networks and ER networks, such as the average degree's effect [10], the improvement of the jamming degree in networks [11], the gradient structure in network synchronization [12], and so on.

Most of the previous work has been focused on the analysis of the jamming degree in unweighed networks, which is not very exact in reality. For example, in metabolic networks, the link weight on chemical reaction represents the amount of flux in this reaction [13]; in airplane networks, the link weight represents the airplane traffic flow between airports [14, 15], which has been found to show relationship with the end-points's degrees; in highway networks, the link weight presents the traffic pressure on the road; in friendship networks, the link weight presents the closeness between two friends. In [], the authors studied the jamming degree in weighted gradient networks, in which the weight is related with the two end-points' degrees in the form of $w_{ij} = (k_i \times k_j)^{\alpha}$, and found that the jamming degree can be minimized at some special value of $\alpha$, then they compared the jamming degree in different substrate networks. Recently people found that in resistor networks, the conductance of a link can be expressed in the form of $exp^{-Ar}$, where $r$ is a random number following the uniform distribution $[0,1]$ [16]. The parameter $A$ controls the disorder of the flow distribution. The larger $A$, the more disordered the distribution will be.

In this work, we investigate the effect of the disorder, i.e., the flow's distribution's disorder on the transportation ability in BA networks and ER networks with the gradient method by the introduction of the weight expressed in the form of $exp^{-Ar}$, where $A$ controls the disorder.

The rest of this paper is organized as follows. Section 2 presents the definition of the jamming coefficient in weighted network; Section 3 briefly reviews the ER network model and the BA network model we used. Section 4 shows the numerical results. Finally, we summarize our work in Sec. 5.

## 2 The definition of the jamming coefficient

Assume we have a network with $N$ nodes, which we call the substrate network. Each node in the substrate network is given a random potential from a uniform distribution, such as $[0,1]$. For node $i$, one can find the neighboring node $j$ that has the highest potential $v_j$ among $i$'s neighbors. A self-loop is also possible if node $i$ has the highest potential among them. Then a gradient network is composed of nodes and the directed edges from source vertex to the vertex of the gradient direction. The jamming coefficient in the network can then be defined as follows:

$$J = 1 - \langle\langle N_{receive}/N_{send}\rangle_v\rangle_{network}, \tag{1}$$

where $\langle ...\rangle_v$ and $\langle ...\rangle_{network}$ denote the average over random potentials and different network configurations, respectively. $J$ means the probability that a randomly selected node has no incoming link. A large value of $J$ means a high level of congestion in the network.

In weighted gradient network, each vertex is assumed to produce a different amount of flow and send them to one of its neighbors with the highest potential. The amount of flow sent by $i$ can be defined as follows:

$$w_{send}(i) = exp(-Ar) \tag{2}$$

where $r$ is the random number taken from the uniform distribution in the range of $[0,1]$, and $A$ controls the disorder of the flow in the network. The larger $A$ is, the more disordered the flow distributes. The *Capacity* of node $i$ can be defined as $C_i = w_{send}(i)$. If the total amount of flow received by $i$, denoted by $\sum_{j\in\Omega(i)} w_{receive(j)}(i)$ is larger than $i$'s capacity $C_i$, then there should remain $(\sum_{j\in\Omega(i)} w_{receive(j)}(i) - C_i)$ amount of flow that can not be processed by vertex $i$. Similarly, the jamming coefficient $J$ in weighted gradient networks can be redefined as follows:

$$J = \langle\langle \frac{\sum_{i=1}^{N}[\sum_{j\in\Omega(i)} w_{receive(j)}(i) - C_i]}{\sum_{i=1}^{N} C_i}\rangle_v\rangle_{network}, \tag{3}$$

where $\Omega(i)$ is the set of vertex $i$'s neighbors. $[x]$ means if $x>0$, $[x]=x$; otherwise, $[x]=0$. The definition of the jamming coefficient indicates that the ratio of the amount of flow not being processed by vertices to the total amount of flow being sent in the network. If all vertices send out the same amount of flow, it will return to the unweighted case.

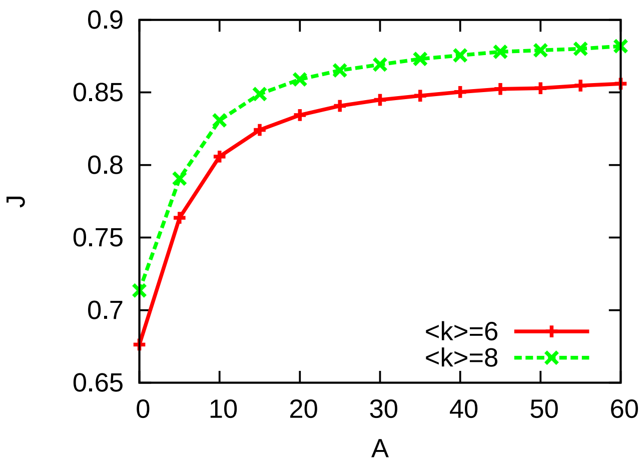


Figure 1: (Color online)The jamming coefficient $J$ versus $A$ in ER networks of the size $N = 1000$ and the average degree $\langle k \rangle = 6,8$. Each point is the average result over 20000 times on 20 network realizations.

# 3 Network model

To compare the jamming degree in BA networks and ER networks with weighted flow distribution, we first briefly review the process of constructing these models. First, we construct ER networks with size $N$ by randomly connecting vertices with $\langle k \rangle \times N/2$ links, where $\langle k \rangle$ is the average connectivity of the network. The only constrained condition is that the final networks should be connected networks. To construct SF networks, we use the basic BA model, whose the degree distribution follows a power-law with degree exponent equal to 3. We produce several networks with different connectivities and then compare the jamming degrees in these networks.

# 4 Numerical results

The jamming coefficient is determined by two elements: the first is the number of neighbors, i.e., the average connectivity and the second is the amount of flow vertex received. In Fig. 1 we show the jamming degree versus parameter $A$ for different average connectivies with $\langle k \rangle = 6,8$, where $A$ ranges from small value 5 to larger value 60, which presents the increase of disorder in the arrangement of the flow distribution.

We can see that with the increase of $A$, the jamming coefficient $J$ increases drastically. This means that the more heterogeneous the flows distribute, the more serious the jamming degree will be. When $A = 0$, all vertices send the same amount of flow, which returns to the case of unweighted one, at this time, the network gets the lowest jamming degree. It can be explained that for large $A$, the magnitudes of the flows are very heterogeneous and there exists few number of flows with much larger magnitude than others. The amount of flows will increase the remaining flows that cannot be processed and cause the network to get jammed.

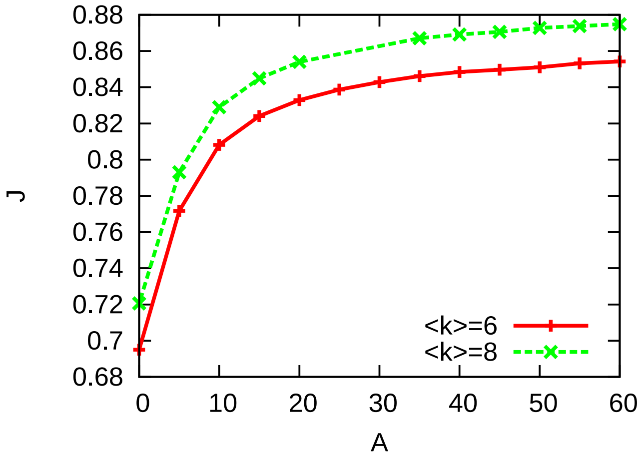


Figure 2: (Color online)The jamming coefficient $J$ versus $A$ in BA networks of the size $N = 1000$ for the average degree $\langle k \rangle = 6, 8$. Each point is the average result over 20000 times on 20 network realizations.

Then, we investigate the jamming degree in BA networks in Fig. 2. We can see that similar to the results in ER networks, BA networks also meet a serious jammed problem with the more disordered distribution of the flow. Specifically, if $A$ is not so large, for example, $A \in [0, 10]$, then the gap between the curves for $\langle k \rangle = 6$ and 8 is not considerable, while with the increase of $A$, the flow distribution becomes disordered, which deduces a large gap between the jamming coefficient. Another observation is that the jamming coefficient $J$ increases with the average connectivity, due to the high probability for vertices to receive the flow from their neighbors.

The above results clearly show that both the network connectivity and the carrying load coaffect the jamming degree of the network. The heterogeneity of the flow distribution will cause the network with a very serious jamming problem. An intuitive way to protect the network from jamming is to arrange vertices with different capacities according to the their degrees, which may improve the network transportation ability and decrease the jammed degree.

## 5 Conclusion and discussion

In this work, we investigate the jamming degree in weighted gradient networks, where the traffic flow distributes in the network heterogeneously or homogeneously depending on the exact values of controlled parameter. The results show that the more heterogeneous the flows distribute, the more serious the network will get jammed. The problem of how to find efficient ways to protect networks from getting jammed deserves a deeper study and discussion in the future.

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